Lecture Notes

09PC602
DIGITAL SIGNAL PROCESSING

VI Sem B.E (I.T)

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IIR Filters Unit-3

Analog Lowpass Filter Design:-

General form of analog filter transfer function is

$$H(S) = \frac{N(S)}{D(S)} = \frac{\sum_{i=0}^{M} a_i s^i}{1 + \sum_{i=1}^{N} b_i s^i}$$

where $H(S)$ is the Laplace transform of $h(t)$.

i.e. $$H(S) = \int_{-\infty}^{\infty} h(t) e^{-st} dt .$$

$N \geq M$ must be satisfied.

Two types of analog filter:

(i) Butterworth filter

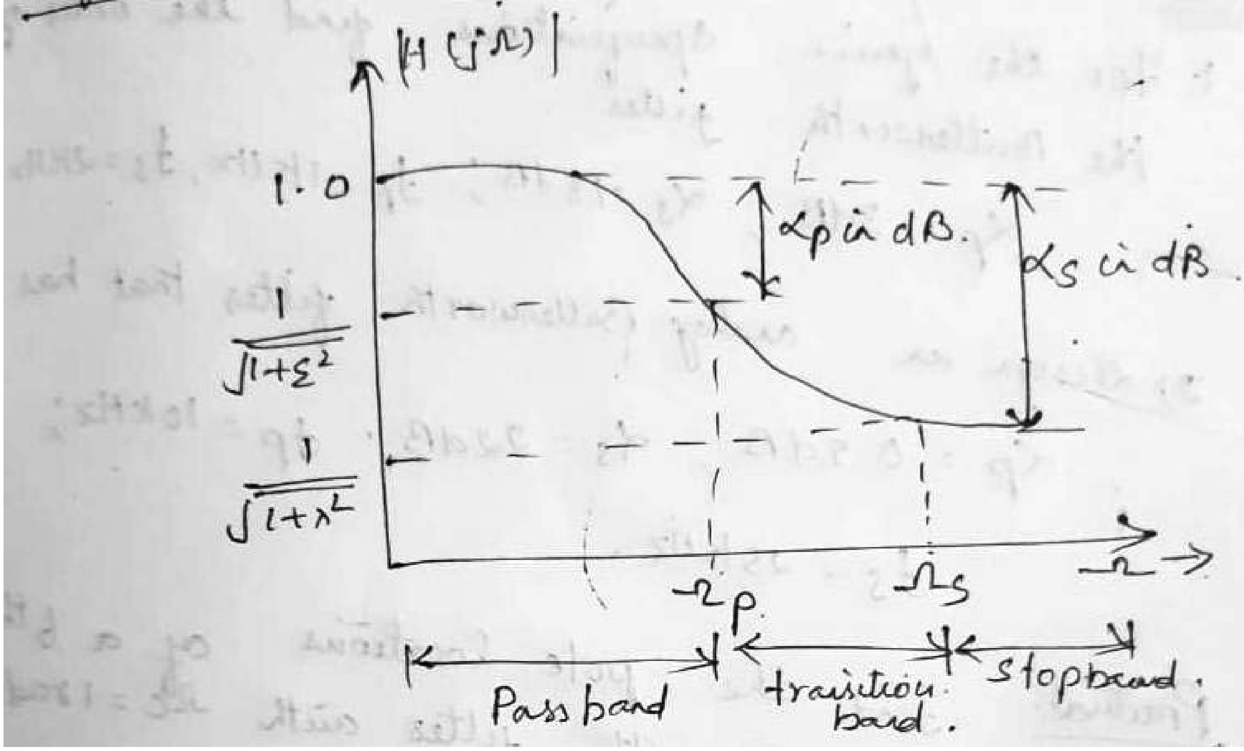
(ii) chebyshev filter.

(i) Butterworth filter:–

Magnitude function is given by

$$|H(j\Omega)| = \frac{1}{\left[1 + \left(\Omega/\Omega_c\right)^{2N}\right]^{1/2}} \qquad N = 1, 2, 3, \ldots$$

Magnitude response:



Steps to design an analog Butterworth lowpass filter:

1. From the given specifications find the order of the filter 'N'.
2. Round off it to the next highest integer.
3. Find the transfer function $H(S)$ for $\Omega_c = 1$ rad/sec for the value of N.
4. Calculate the value of cut off frequency Ω_c.
5. Find the transfer function $H_a(s)$ for the above value of Ω_c by substituting $S \Rightarrow \frac{S}{\Omega_s}$ in $H(S)$.

III unit **IIR Filter.**

1. Given Specifications $\alpha_p = 1dB$, $\alpha_s = 30dB$, $\Omega_p = 200$ rad/sec, $\Omega_s = 600$ rad/sec. determine the order of the filter (N).

Given $\alpha_p = 1\,dB$; $\Omega_p = 200$ rad/sec

$\alpha_s = 30\,dB$; $\Omega_s = 600$ rad/sec.

$$N \geq \frac{\log \sqrt{\frac{(10)^{0.1\alpha_s} - 1}{(10)^{0.1\alpha_p} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log \sqrt{\frac{(10)^{0.1(30)} - 1}{(10)^{0.1(1)} - 1}}}{\log\left(\frac{600}{200}\right)} = \frac{\log \sqrt{\frac{1000 - 1}{1.259 - 1}}}{\log(3)}$$

$$\geq \frac{\log \sqrt{\frac{999}{0.259}}}{\log(3)} \geq \frac{\log \sqrt{3857.14}}{\log(3)}$$

$$\geq \frac{\log(62.11)}{\log(3)}$$

$$\geq \frac{1.793}{0.477}$$

$$\geq 3.75$$

$$\boxed{N \approx 4}$$

H.W.

2) Determine the order of LP butterworth filter it has 3db pass band attenuation at 500Hz. 40db at 1000 Hz stop band attenuation

Given: $f_p = 500\,Hz \Rightarrow \Omega_p = 2\pi f_p$

$= 2 \times \pi \times 500$

$= 1000\pi$ r/s

$\alpha_p = 3dB$

$f_s = 1000\,Hz \Rightarrow \Omega_s = 2\pi f_s$

$= 2 \times \pi \times 1000$

$= 2000\pi$ r/s

$\alpha_s = 40dB$

$$N \geq \frac{\log \sqrt{\dfrac{(10)^{0.1(\alpha s)} - 1}{(10)^{0.1(\alpha p)} - 1}}}{\log \left(\dfrac{\Omega s}{\Omega p}\right)}$$

$$\geq \frac{\log \sqrt{\dfrac{(10)^{0.1(40)} - 1}{(10)^{0.1(3)} - 1}}}{\log \left(\dfrac{2000\pi}{1000\pi}\right)}$$

$$\geq \frac{\log \sqrt{\dfrac{(10)^{4} - 1}{10^{0.3} - 1}}}{\log (2)} \geq \frac{\log \sqrt{\dfrac{9999}{0.995}}}{\log (2)}$$

$$\geq \frac{\log (99.9949)}{\log (2)} \geq \frac{1.999}{0.301}$$

$$\geq 6.64 \quad \therefore N \approx 6.6$$

order of $\boxed{N \approx 7}$

3) Design an analog butterworth filter has 2dB pass band attenuation at a freq of 20 rad/sec & atleast 10dB stop band attenuation at 30 r/s.

Given: $\Omega_p = 20 \text{ rad/sec}$; $\alpha_p = 2\text{dB}$
$\Omega_s = 30 \text{ rad/sec}$. $\alpha_s = 10\text{dB}$.

(Low Pass filter:)

Step 1: To find order of filter (N).

$$N \geq \frac{\log \sqrt{\frac{(10)^{0.1\alpha_s} - 1}{(10)^{0.1\alpha_p} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log \sqrt{\frac{(10)^{0.1(10)} - 1}{(10)^{0.1(2)} - 1}}}{\log\left(\frac{30}{20}\right)}$$

$$N \geq \frac{\log \sqrt{\frac{(10)^{1} - 1}{(10)^{0.2} - 1}}}{\log(3/2)}$$

$$N \geq \frac{\log\sqrt{\frac{9}{0.583}}}{\log(3/2)}$$

$$\geq \frac{\log\sqrt{15.437}}{\log(1.5)}$$

$$\geq \frac{\log(3.929)}{\log(1.5)}$$

$$\geq \frac{0.594}{0.176}$$

$$\geq 3.375$$

$$\geq 3.4$$

$$\boxed{N \approx 4}$$ $\therefore$ $\boxed{\text{Order of } N = 4.}$

Step2: To find H(s) at $\Omega c = 1$ rad/sec

Here $N = 4$.

$$\therefore H(s) = \prod_{k=1}^{N/2} \frac{1}{s^2 + b_k s + 1}.$$

$$= \prod_{k=1}^{4/2} \frac{1}{s^2 + b_k s + 1}$$

$$= \prod_{k=1}^{2} \frac{1}{s^2 + b_k s + 1}$$

$$= \frac{1}{s^2 + b_1 s + 1} + \frac{1}{s^2 + b_2 s + 1}.$$

$$= \frac{1}{s^2 + 0.7653s + 1} \times \frac{1}{s^2 + 1.8477s + 1}$$

$b_k = 2\sin\left(\frac{2k-1}{2N}\right)\pi$

When $k = 1$

$= 2\sin\left(\frac{2\times1-1}{2\times4}\right)\pi$

$= 2\sin\left(\frac{1}{8}\right)\pi$

$= 2\sin(\pi/8)$

$= 0.7653$

$k = 2$

$b_2 = 2\sin\left(\frac{2\times2-1}{2\times4}\right)\pi$

$= 2\sin\left(\frac{3\pi}{8}\right)$

$= 1.8477$

Step3 : To find cut off frequency:-

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}}$$

$$\Omega_c = \frac{20}{(10^{(0.1)\times 2} - 1)^{\frac{1}{2\times 4}}}$$

$$= \frac{20}{(10^{(0.2)} - 1)^{\frac{1}{8}}} = \frac{20}{(0.5848)^{1/8}}$$

$$= \frac{20}{0.9351}$$

$$\boxed{\Omega_0 = 21.386 \text{ rad/sec}}$$

Step4: Analog transfer function:

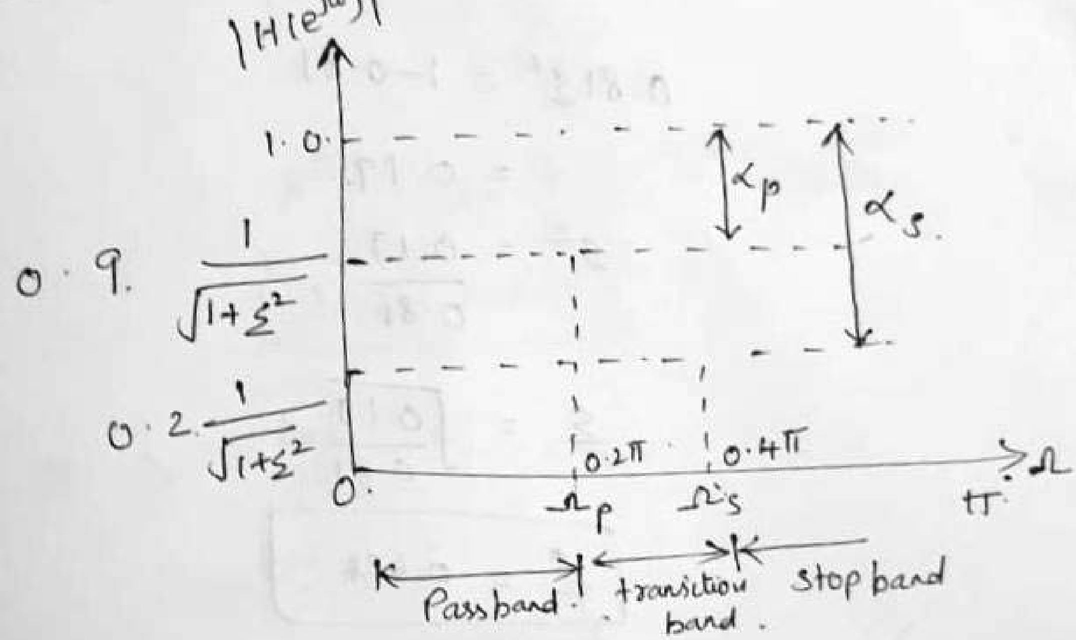
$$Ha(s) = H(s)\Big|_{s \to \frac{s}{\Omega_c}} = \frac{s}{21.3868}$$

$$Ha(s) = \frac{1}{\left[\left(\frac{s}{21.3868}\right)^2 + 0.7653\left(\frac{s}{21.3868}\right) + 1\right]\left\{\left(\frac{s}{21.3868}\right)^2 + 1.8477\left(\frac{s}{21.3868}\right) + 1\right\}}$$

$$Ha(s) = \frac{1}{\frac{s^2 + 16.3686s + 457.395}{457.395} \times \frac{s^2 + 39.51s + 457.395}{457.395}}$$

$$Ha(s) = \frac{0.20921 \times 10^6}{(s^2 + 16.3686s + 457.395)(s^2 + 39.51s + 457.395)}$$

3) Consider the response of Low pass Filter Butterworth magnitude response



For the given specification design an analog butterworth filter $0.9 \leq |H(j\Omega)| \leq$ for $0 \leq \Omega \leq 0.2\pi$

$0 \leq \Omega \leq \pi$.

and $|H(j\Omega)| \leq 0.2$ for $0.4\pi \leq \Omega \leq \pi$.

(i) To find ε: (To remove sq. root, Square both sides)

$$(0.9)^2 = \left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)^2$$

$$0.81 = \frac{1}{1+\varepsilon^2}$$

$$0.81(1+\varepsilon^2) = 1$$

$$0.81 + 0.81\xi^2 = 1$$

$$0.81\xi^2 = 1 - 0.81$$

$$= 0.19.$$

$$\xi^2 = \frac{0.19}{0.81}$$

$$\xi = \sqrt{\frac{0.19}{0.81}}.$$

$$\boxed{\xi = 0.484.}$$

$$0.2 = \frac{1}{\sqrt{1+\lambda^2}}.$$

Square both sides.

$$(0.2)^2 = \left(\frac{1}{\sqrt{1+\lambda^2}}\right)^2.$$

To find λ

$$0.04 = \frac{1}{1+\lambda^2}.$$

$$0.04(1+\lambda^2) = 1$$

$$0.04 + 0.04\lambda^2 = 1$$

$$0.04\lambda^2 = 0.96$$

$$\lambda^2 = \frac{0.96}{0.04}$$

$$\lambda^2 = (24) \Rightarrow \lambda = \sqrt{24}.$$

$$\boxed{\lambda = 4.898}$$

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log\left(\frac{\Omega s}{\Omega p}\right)}$$

$$\geq \frac{\log\left(\frac{4.898}{0.484}\right)}{\log\left(\frac{0.4\pi}{0.2\pi}\right)}$$

$$\geq \frac{\log(10.1198)}{\log(2)}$$

$$\geq \frac{1.005}{0.3010}$$

$$N \geq 3.3$$

$$\boxed{N \simeq 4}$$

$$H(S) = \frac{1}{(S^2+0.7653S+1)(S^2+1.8477S+1)}$$

when $N=4$,

$$H(S) = \prod_{k=1}^{N/2} \frac{1}{S^2+b_kS+1}$$

$$= \prod_{k=1}^{2} \frac{1}{S^2+b_kS+1}$$

$$b_k = 2\sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$b_1 = 2\sin(\pi/8) = 0.7653$$

$$b_2 = 2\sin(3\pi/8)$$

$$b_2 = 1.8477$$

$$H(S) = \frac{1}{S^2+0.7653S+1} \times \frac{1}{S^2+1.8477S+1}$$

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}}$$

here. $\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}}$, $\varepsilon\left(10^{0.1\alpha_p}-1\right)^{1/2}$.

$$\Omega_c = \frac{0.2\pi}{(0.484)^{1/4}}$$

$$= \frac{0.2\pi}{0.8340}$$

$$\boxed{\Omega_c = 0.24\pi}$$

$$H_a(s) = H(s)\Big|_{s \rightarrow \frac{s}{0.24\pi}} \quad \text{or} \quad \frac{s}{\Omega_c}$$

$$H_a(s) = \frac{1}{\left(\frac{s^2+0.577s+0.576\pi^2}{0.0576\pi^2}\right)^2 \left\{\frac{s^2+1.393s+0.576\pi^2}{0.0576\pi^2}\right\}}$$

$$\boxed{H_a(s) = \frac{0.323}{(s^2+0.577s+0.576\pi^2)(s^2+1.393s+0.0576\pi^2)}}$$

AW

1. For the given specifications find the order of the Butterworth filter.

$\alpha_p = 3dB$; $\alpha_s = 18dB$; $f_p = 1kHz$, $f_s = 2kHz$.

2) Design an analog Butterworth filter that has

$\alpha_p = 0.5dB$; $\alpha_s = 22dB$; $f_p = 10kHz$;

$f_s = 25kHz$.

Practical: Find the pole locations of a 6th order Butterworth filter with $\Omega_c = 1 rad/sec$.

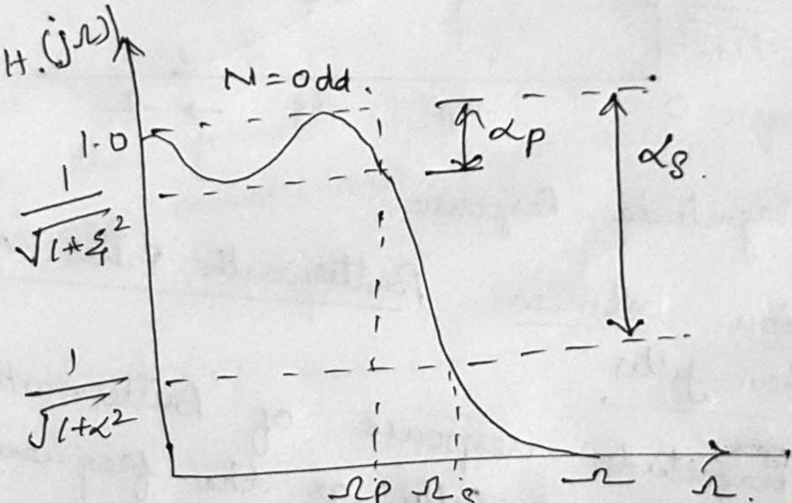
Analog lowpass Chebyshev Filters:

Two types:

(i) Type I chebyshev

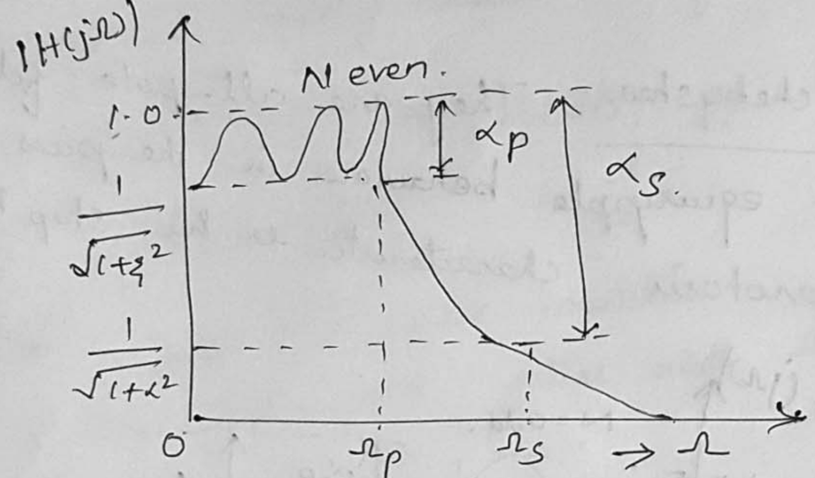
(ii) Type II chebyshev.

Type I chebyshev:- They are all-pole filters that exhibit equiripple behaviour in the pass band and monotonic characteristic in the stop band.



Magnitude response.

Type II chebyshev:- contains both poles and zeros that exhibit a monotonic behaviour in the pass band and an equiripple behaviour in the stopband.



Magnitude Response.

Comparison between Butterworth Filter and chebyshev filter:

1) The magnitude response of Butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞, whereas the magnitude response of the chebyshev filter exhibits ripples in the passband or stopband according to the type.

2) The transition band is more in Butterworth filter when compared to chebyshev filter.

3) The poles of the Butterworth filter lie on a circle, poles of the chebyshev lie on an ellipse

4) The number of poles in Butterworth are more, whereas for chebyshev, the poles are less.

Steps to design an analog Chebyshev lowpass filter:

1. From the given specifications find the order of the filter N.
2. Round off it to next higher integer.
3. Using the formula find the values of a and b, which are minor and major axis repectively.

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right]$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right]$$

where $\mu = \varepsilon^{-1} + \sqrt{\varepsilon^{-2}+1}$

$\varepsilon = \sqrt{10^{0.1\alpha_p} - 1}$

Ω_p = Passband frequency.

α_p = Maximum allowable attenuation in the passband.

(∵ for normalized chebyshev filter $\Omega_p = 1$ rad/sec)

4) calculate the poles of chebyshev filter which lie on an ellipse by using the formula.

$$S_k = a \cos \phi_k + jb \sin \phi_k \quad k = 1, 2, \cdots, N$$

where
$$\phi_k = \frac{\pi}{2} + \left(\frac{2k-1}{2N}\right)\pi \quad k = 1, 2, \cdots, N$$

5) Find the denominator polynomial of the transfer function using the above poles.

6) The numerator of the transfer function depends on the value of N.

a) For N odd substitute $S = 0$ in the denominator polynomial and find the value. This value is equal to the numerator of the transfer function.

(∵ For N odd the magnitude response $|H(j\Omega)|$ starts at 1)

(b) For N even substitute $S = 0$ in the denominator polynomial and divide the result by $\sqrt{1+\varepsilon^2}$. This value is equal to the numerator.

1) Given the specifications $\alpha_p = 3dB$; $\alpha_s = 16dB$; $f_p = 1kHz$ and $f_s = 2kHz$. Determine the order of the filter using chebyshev approximation. Find $H(s)$

Given: $\Omega_p = 2\pi f_p$

$= 2\pi \times 1 \times 1000$

$= 2000\pi$ rad/sec

$\Omega_s = 2\pi f_s$

$= 2\pi \times 2 \times 1000$

$= 4000\pi$ rad/sec.

$\alpha_p = 3dB$

$\alpha_s = 16dB$.

Step 1:

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{(10)^{0.1\alpha_s}-1}{(10)^{0.1\alpha_p}-1}}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}}$$

$$= \frac{\cosh^{-1}\sqrt{\frac{(10)^{1.6}-1}{(10)^{0.3}-1}}}{\cosh^{-1}\left(\frac{4000\pi}{2000\pi}\right)} = 1.91$$

Step 2: Rounding N to next highest value $N = 2$.

For 'N' even, the oscillatory curve starts from

$$\frac{1}{\sqrt{1+\epsilon^2}}.$$

Step3: The values of minor axis and major axis can be found as below:

$$\varepsilon = (10^{0.1\alpha p} - 1)^{0.5} = ((10)^{0.3} - 1) = 1.$$

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}} = 2.414.$$

$$a = \Omega p \frac{[\mu^{1/N} - \mu^{-1/N}]}{2}.$$

$$= 2000\pi \frac{[(2.414)^{1/2} - (2.414)^{-1/2}]}{2}$$

$$= 910\pi$$

$$b = \Omega p \frac{[\mu^{1/N} + \mu^{-1/N}]}{2}$$

$$= 2000\pi \frac{[(2.414)^{1/2} + (2.414)^{-1/2}]}{2} = 2197\pi$$

Step4: The poles are given by

$$s_k = a\cos\phi_k + jb\sin\phi_k, \quad k = 1, 2.$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \qquad k = 1, 2$$

$$\phi_1 = \pi/2 + \pi/4 = 135^\circ$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = 225^\circ$$

$$s_1 = a\cos\phi_1 + jb\sin\phi_1 = -643.46\pi + j1554\pi$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = -643.46\pi - j1554\pi .$$

Step 5: The denominator of H(S):

$$H(S) = (S + 643.46\pi)^2 + (1554\pi)^2$$

Step 6: The numerator of H(S):

$$H(S) = \frac{(643.6\pi)^2 + (1554\pi)^2}{\sqrt{1+\xi^2}} = (1414.38)^2\pi^2.$$

The transfer function $H(S) = \dfrac{(1414.38)^2\pi^2}{S^2 + 1287\pi S + (1682)^2\pi^2}$

H.W.:

1) Design a chebyshev filter with maximum passband attenuation of 2.5 dB at $\Omega_p = 20$ rad/sec and the stop band attenuation of 30 dB at $\Omega_s = 50$ rad/sec.

IIR Digital Filter :-

(i) Approximation of derivatives

(ii) Impulse invariant transformation.

(iii) The bilinear transformation.

(iv) The matched z- transformation.

Steps to design digital filter using impulse invariant method:

(i) For the given specifications, $\Omega_p = \omega_p/T$, $\Omega_s = \omega_s/T$, find $H_a(s)$

(ii) Select the sampling rate of the digital filter T seconds per sample.

(iii) Express the analog transfer function as the "sum of single pole filters" by using partial fraction.

$$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}.$$

(iv) Compute the z-transform of the digital filter by using the formula

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

for high sampling rates (for small T) use,

$$H(Z) = \sum_{K=1}^{N} \cdot \frac{TC_k}{1-e^{P_K T} z^{-1}}$$

1) For a analog transfer function $H(s) = \frac{2}{(s+1)(s+2)}$ determine H(Z) using impulse invariance method assume T = 1 sec.

Given :- $H(s) = \frac{2}{(s+1)(s+2)}$; T = 1 sec

$\frac{2}{(s+1)(s+2)} \Rightarrow$ $A = H(s)\big|_{s=-1}$

$A = \frac{2}{(s+1)(s+2)}\bigg|_{s=-1}$

$\boxed{A = -2}$ & $\boxed{B = -2}$.

~~A+B = -2~~ $0 = 0A$.

$$H(s) = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$2 = A(s+2) + B(s+1).$$

$A + s = -2$; at $s = -1$

$\boxed{B = -2}$ $\boxed{2 = A}$

$$H(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$= \frac{2}{s+1} - \frac{2}{s+2}$$

$$= \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

Using impulse invariance technique, if

$$H(s) = \sum_{k=1}^{N} \frac{C_k}{s-P_k} \text{ then}$$

i.e. $$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$$

i.e. $(s-P_k)$ is transformed to ~~$1-e^{P_k T} z^{-1}$~~ $1-e^{P_k T} z^{-1}$.

Two poles $P_1 = -1$ & $P_2 = -2$ So

$$H(s) = \frac{2}{1-e^{-T} z^{-1}} - \frac{2}{1-e^{-2T} z^{-1}}$$

For $T = 1$ sec

$$H(z) = \frac{2}{1-e^{-1} z^{-1}} - \frac{2}{1-e^{-2} z^{-1}}$$

$$= \frac{2}{1-0.3678 z^{-1}} - \frac{2}{1-0.1353 z^{-1}}$$

$$\boxed{H(z) = \frac{0.465 z^{-1}}{1-0.503 z^{-1} + 0.04976 z^{-2}}}$$

3) Using impulse invariance with T = 1 sec, determine $H(z)$ if $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$.

Given: $H(s) = \frac{1}{s^2+\sqrt{2}s+1}$

$$h(t) = L^{-1} H(s)$$

$$= L^{-1}\left[\frac{1}{s^2+\sqrt{2}s+1}\right]$$

$$= L^{-1}\left[\frac{1}{(s+\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2}\right]$$

$$= L^{-1}\left[\sqrt{2}\ \frac{\frac{1}{\sqrt{2}}}{[s+\frac{1}{\sqrt{2}}]^2+(\frac{1}{\sqrt{2}})^2}\right]$$

$$= \sqrt{2}\, L^{-1}\left[\frac{\frac{1}{\sqrt{2}}}{(s+\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2}\right]$$

$$= \sqrt{2}\, e^{-t/\sqrt{2}} \sin(t/\sqrt{2}).$$

Let $t = nT$

$$h(nT) = \sqrt{2}\, e^{-nT/\sqrt{2}} \sin \frac{nT}{\sqrt{2}}$$

If $T = 1$ sec

$$h(n) = \sqrt{2}\, e^{-n/\sqrt{2}} \sin \frac{n}{\sqrt{2}}$$

$$H(z) = Z\,[h(n)]$$

$$= \sqrt{2} \left[\frac{e^{-1/\sqrt{2}} z^{-1} \sin 1/\sqrt{2}}{1 - 2e^{-1/\sqrt{2}} z^{-1} \cos \frac{1}{\sqrt{2}} + e^{-\sqrt{2}} z^{-2}} \right]$$

$$H(z) = \frac{0.453 z^{-1}}{1 - 0.7497 z^{-1} + 0.2432 z^{-2}}$$

//.

4) An analog filter has a transfer function $H(s) = \frac{10}{s^2 + 7s + 10}$. Design a digital filter equivalent to this using impulse invariant method for $T = 0.2$ sec.

Given $H(s) = \frac{10}{s^2 + 7s + 10}$.

$$= \frac{-3.33}{s+5} + \frac{3.33}{s+2}$$

$$= \frac{-3.33}{s-(-5)} + \frac{3.33}{s-(-2)}$$

$$H(z) = T\left[\frac{-3.33}{1-e^{-5T}z^{-1}} + \frac{3.33}{1-e^{-2T}z^{-1}}\right]$$

$$= 0.2\left[\frac{-3.33}{1-e^{-1}z^{-1}} + \frac{3.33}{e^{-0.4}z^{-1}}\right]$$

$$= \left[\frac{-0.666}{1-0.3678z^{-1}} + \frac{0.666}{1-0.67z^{-1}}\right]$$

$$\boxed{H(z) = \frac{0.2012z^{-1}}{1-1.0378z^{-1}+0.2472z^{-2}}}$$ Ans.

H.W.

1) An analog filter has a transfer function

$$H(s) = \frac{5}{s^3+6s^2+11s+6}.$$

Design a digital filter equivalent to this using impulse invariant method for T = 1 sec.

2). An analog ~~transfer~~ ~~function~~ filter has a transfer function

$$H(s) = \frac{s+3}{s^2+6s+25}.$$

Design a digital filter equivalent to this using impulse invariant method for T = 1 Sec.

$$H(s) = \frac{2}{s-(-1)} - \frac{2}{s-(-2)}$$

where $A = 2$; $B = -2$.

$P_1 = -1$; $P_2 = -2$.

$$H(z) = \sum_{k=1}^{N} \frac{C_k}{1-e^{P_k T} z^{-1}}$$

$$= \frac{2}{1-e^{-T}z^{-1}} - \frac{2}{1-e^{-2T}z^{-1}}$$

Substitute $T = 1$ sec.

$$H(z) = \frac{2}{1-e^{-1}z^{-1}} - \frac{2}{1-e^{-2T}z^{-1}}.$$

$$= \frac{2(1-e^{-2}z^{-1}) - 2(1-e^{-1}z^{-1})}{(1-e^{-1}z^{-1})(1-e^{-2}z^{-1})}.$$

$$= \frac{2 - 2e^{-2}z^{-1} - 2 + 2e^{-1}z^{-1}}{1-e^{-2}z^{-1} - e^{-1}z^{-1} + e^{-1}e^{-2}z^{-2}}.$$

$$= \frac{-0.270z^{-1} + 0.735z^{-1}}{1 - 0.135z^{-1} - 0.367z^{-1} + (0.367)(0.135)z^{-2}}$$

$$H(z) = \frac{0.465z^{-1}}{1 - 0.503z^{-1} + 0.04954z^{-2}}$$ //

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2) Analog filter has a transfer function of $H(s) = \frac{10}{s^2+7s+10}$ design a digital filter equivalent to this using impulse invariance method for $T = 0.2$ sec.

$$H(s) = \frac{10}{s^2+7s+10}$$

$$= \frac{10}{(s+2)(s+5)}$$

$$H(s) = \frac{A}{(s+2)} + \frac{B}{(s+5)}$$

$$10 = A(s+5) + B(s+2)$$

at $s = -5$; $10 = -3B \Rightarrow \boxed{B = -3.3}$

at $s = -2$; $10 = A(-2+5)$

$10 = 3A$

$\boxed{A = \frac{10}{3} = 3.3}$

$$H(s) = \frac{3.3}{s-(-2)} - \frac{3.3}{s-(-5)}$$

where
$c_1 = 3.3$
$c_2 = -3.3$
$p_1 = -2$
$p_2 = -5$.

$$H(z) = \sum_{k=1}^{2} \frac{T c_k}{1 - e^{p_k T} z^{-1}}$$

$$= \frac{3.3}{1-e^{-2T}z^{-1}} - \frac{3.3}{1-e^{-5T}z^{-1}}.$$

$$= \frac{3.3}{1-e^{-0.4}z^{-1}} - \frac{3.3}{1-e^{-1}z^{-1}}.$$

$$= \frac{3.3(1-e^{-1}z^{-1}) - 3.3(1-e^{-0.4}z^{-1})}{(1-e^{-0.4}z^{-1})(1-e^{-1}z^{-1})}$$

$$= \frac{3.3 - 3.3e^{-1}z^{-1} - 3.3 + 3.3e^{-0.4}z^{-1}}{1-e^{-1}z^{-1} - e^{-0.4}z^{-1} + e^{-0.4}e^{-1}z^{-2}}$$

$$= \frac{-1.214z^{-1} + 2.212z^{-1}}{1 - 0.3678z^{-1} - 0.6703z^{-1} + 0.2465z^{-2}}$$

$$H(z) = \frac{0.2012z^{-1}}{1 - 1.03810z^{-1} + 0.2465z^{-2}}.$$

Frequency transformation in Analog Domain.

Frequency transformations
- can be used to design lowpass filters with different passband frequencies, high pass filter, bandpass filters and bandstop filters from a normalized lowpass filter ($\Omega_c = 1$ rad/sec).

1. Low pass to Low pass
2. Low pass to high pass
3. Low pass to Band pass
4. Low pass to Band stop.

Design of IIR filters from analog filters:

- Several methods can be used to design digital filters having an infinite duration unit sample response.

- The techniques are all based on converting an analog ~~filter~~ filter into a digital filter.

It posses the following properties

1. The $j\Omega$ - axis in the s-plane should map into the unit circle in the z-plane. Thus there will be a direct relationship between the two frequency variables in the two domains.

2. The left-half plane of the s-plane should map into the inside of the unit circle in the z-plane. Thus a stable analog filter will be converted to a stable digital filter.

<u>Four methods</u> are widely used for digitizing analog filter into a digital filter.

1) Approximation of derivatives.
2) The impulse invariant transformation.
3) The bilinear transformation.
4) The matched z-transformation Technique.

<u>Impulse invariant transformation:</u>

Let $Ha(s)$ is the system function of an analog filter. This can be

$$Ha(s) = \sum_{K=1}^{N} \frac{C_K}{s - P_K} \quad \text{— (1).}$$

Where $\{P_K\}$ → Poles of the analog filter.

$\{C_K\}$ → Coefficients in the partial fraction expansion. (contd)

The inverse Laplace transform of (1) is

$$h_a(t) = \sum_{k=1}^{N} C_k e^{P_k t} \quad t \geq 0 \qquad \text{--- (2)}$$

Sample (2) $h_a(t)$ periodically at $t = nT$, then.

$$h(n) = h_a(nT) = \sum_{k=1}^{N} C_k e^{P_k nT} \qquad \text{--- (3)}$$

W.k. $$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n} \qquad \text{--- (4)}$$

Substitute (3) in (4) then

$$H(z) = \sum_{n=0}^{\infty} \sum_{k=1}^{N} C_k e^{P_k nT} z^{-n}$$

$$= \sum_{k=1}^{N} C_k \sum_{n=0}^{\infty} \left(e^{P_k T} z^{-1}\right)^n$$

$$= \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

i.e $$H_a(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}$$

then

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}.$$

For high sampling rate (for small T) the digital filter gain is high.

$$\therefore \boxed{H(z) = \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k T} z^{-1}}}$$

Note: Due to the presence of aliasing, the impulse invariant method is appropriate for the design of lowpass and bandpass filters only. where it is unsuccessful for implementing digital filters such as a high pass filter.

Steps to design a digital filter using Impulse Invariance method

1) For the given specifications, find $H_a(s)$, the transfer function of an analog filter.

2) Select the sampling rate of the digital filter, T seconds per sample.

3) Express the analog filter transfer function as the sum of single - pole filters.

$$H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}.$$

4.) compute the z- transform of the digital filter by using the formula

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}$$

For high dampling rates use

$$H(z) = \sum_{k=1}^{N} \frac{T c_k}{1 - e^{p_k T} z^{-1}}.$$

4) Apply impulse invariant method and find $H(z)$ for $H(s) = \frac{s+a}{(s+a)^2+b^2}$.

Soln: The inverse laplace transform of given function is

$$h(t) = \begin{cases} e^{-at}\cos(bt) & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$h(nT) = \begin{cases} e^{-anT}\cos(bnT) & \text{for } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$H(z) = \sum_{n=0}^{\infty} e^{-anT} \cos(bnT) z^{-n}.$$

$$= \sum_{n=0}^{\infty} \left[e^{-anT} z^{-n} \left[\frac{e^{jbnT} + e^{-jbnT}}{2} \right] \right]$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \left[(e^{-(a-jb)T} z^{-1})^n + (e^{-(a+jb)T} z^{-1})^n \right]$$

$$= \frac{1}{2} \left[\frac{1}{1-e^{-(a-jb)T} z^{-1}} + \frac{1}{1-e^{-(a+jb)T} z^{-1}} \right]$$

$$H(z) = \frac{1 - e^{-aT} \cos(bT) z^{-1}}{1 - 2e^{-aT} \cos(bT) z^{-1} + e^{-2aT} z^{-2}}$$

Design of IIR filter using Bilinear Transformation

- is a conformal mapping one.
- transforms 'jΩ' axis into the unit circle in the z-plane.
- Avoiding aliasing of frequency components.
- all points in the LHP of 's' are mapped inside the unit circle in the z-plane.
- all points in the RHP of 's' are mapped into corresponding points outside the unit circle in the z-plane.

Proof:

Let us consider an analog linear filter with transfer function.

$$H(s) = \frac{b}{s+a} \quad \text{— (1)}$$

can be written as

$$\frac{Y(s)}{X(s)} = \frac{b}{s+a} \quad \text{— (2)}$$

ie. $Y(s)(s+a) = bX(s)$. — (3)

$$sY(s) + aY(s) = bX(s).$$

This can be characterized by the diff. equation

$$\frac{dy(t)}{d(t)} + a\,y(t) = b\,x(t) \quad — (4)$$

y(t) can be approximated by

$$y(t) = \int_{t_0}^{t} y'(\tau)\,d\tau + y(t_0). \quad — (5)$$

where y'(t) denotes the derivative of y(t).

Substitute $t = nT$ & $t_0 = nT - T$.

$$y(nT) = \frac{T}{2}\left[y'(nT) + y'(nT-T)\right] + y(nT-T) \quad — (6).$$

From (4) we obtain

$$y'(nT) = -a\,y(nT) + b\,x(nT) \quad — (7).$$

Substitute (7) in (6) we get.

$$y(nT) = \frac{T}{2}\left[-a\,y(nT) + b\,x(nT) + b\,x(nT) - a\,y(nT-T) + b\,x(nT-T)\right] + y(nT-T)$$

$\Rightarrow$

$$y(nT) + \frac{aT}{2} y(nT) - \left[1 - \frac{aT}{2}\right] y(nT - T) =$$

$$\frac{bT}{2}\left[x(nT) + x(nT - T)\right]. \quad \text{--- (8)}$$

With $y(n) = y(nT)$ & $x(n) = x(nT)$ we obtain

$$\left[1 + \frac{aT}{2}\right] y(n) - \left[1 - \frac{aT}{2}\right] y(n-1) = \frac{bT}{2}\left[x(n) - x(n-1)\right] \quad \text{--- (9)}$$

The Z-transform of the eqn. (9) is

$$\left[1 + \frac{aT}{2}\right] Y(z) - \left[1 - \frac{aT}{2}\right] z^{-1} Y(z) = \frac{bT}{2}\left[1 + z^{-1}\right] X(z)$$

The system function of the digital filter is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{bT}{2}(1 + z^{-1})}{1 + \frac{aT}{2} - \left[1 - \frac{aT}{2}\right] z^{-1}}$$

$$= \frac{\frac{bT}{2}(1 + z^{-1})}{(1 - z^{-1}) + \frac{aT}{2}(1 + z^{-1})}$$

Dividing Nr/.. & Dr/.. by $T/2(1+z^{-1})$ we get

$$H(z) = \frac{b}{\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+a} \quad \text{—} \quad (9)$$

comparing eqn (2) & (9) the mapping from s-plane to the z-plane can be

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right] \quad \text{—} \quad (10)$$

The relationship between s and z is known as bilinear transformation.

Let $z = re^{j\omega}$.

$s = \sigma + j\Omega$.

Eqn/.. (10) can be expressed as.

$$s = \frac{2(z-1)}{T(z+1)}$$

$$= \frac{2}{T}\left[\frac{re^{j\omega}-1}{re^{j\omega}+1}\right] = \frac{2}{T}\left[\frac{r\cos\omega - 1 + jr\sin\omega}{r\cos\omega + 1 + jr\sin\omega}\right]$$

$$= \frac{2}{T}\left[\frac{r\cos\omega - 1 + jr\sin\omega}{r\cos\omega + 1 + jr\sin\omega}\right]\left[\frac{r\cos\omega + 1 - jr\sin\omega}{r\cos\omega + 1 - jr\sin\omega}\right]$$

$$= \frac{2}{T}\left[\frac{r^2\cos^2\omega - 1 + r^2\sin^2\omega + j2r\sin\omega}{(r\cos\omega + 1)^2 + r^2\sin^2\omega}\right]$$

$$= \frac{2}{T}\left[\frac{r^2\cos^2\omega - 1 + r^2\sin^2\omega + j2r\sin\omega}{1 + r^2\cos^2\omega + 2r\cos\omega + r^2\sin^2\omega}\right]$$

Separating real and imaginary parts,

$$s = \frac{2}{T}\left[\frac{r^2 - 1}{1 + r^2 + 2r\cos\omega} + j\,\frac{2r\sin\omega}{1 + r^2 + 2r\cos\omega}\right] \quad \text{—— (11)}$$

$$\sigma = \frac{2}{T}\left[\frac{r^2 - 1}{1 + r^2 + 2r\cos\omega}\right];$$

$$\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{1 + r^2 + 2r\cos\omega}\right] \quad \text{—— (12)}$$

Case(i):- if $r < 1$ then $\sigma < 0$.

$\therefore$ LHP in 's' maps into the inside of the unit circle in the z-plane.

Case(ii) if $r > 1$, then $\sigma > 0$.

$\therefore$ RHP in the 's' maps into the outside of the unit circle.

Case(iii) if $r = 1$, then $\sigma = 0$.

and

$$\Omega = \frac{2}{T} \frac{\sin\omega}{1+\cos\omega} = \frac{2}{T} \frac{2\sin\frac{\omega}{2}\cos\frac{\omega}{2}}{2\cos^2\frac{\omega}{2}}$$

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

(or)

$$\omega = 2\tan^{-1}\frac{\Omega T}{2}.$$

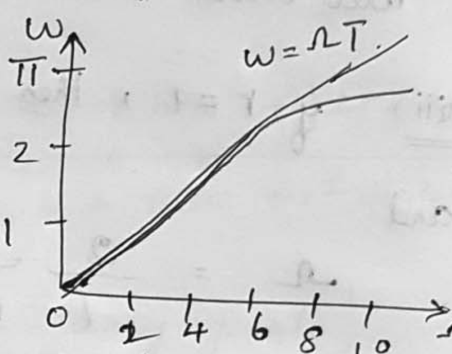
//

Warping effect:- The relation between the analog and digital frequencies in bilinear transformation is given by $\Omega = \frac{2}{T}\tan\frac{\omega}{2}$

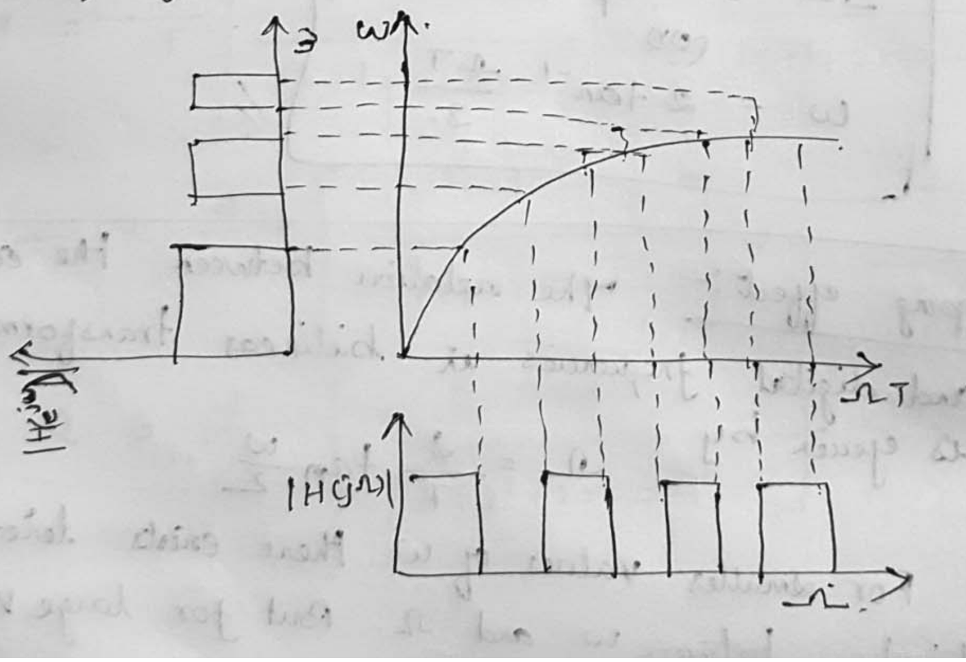
For smaller values of ω there exists linear relationship between ω and Ω. But for large values of

The relationship is non-linear. This non-linearity introduces distortion in the frequency axis. This is known as <u>warping effect</u>. This effect compresses the magnitude and phase response at high frequencies.

<u>Relationship between Ω and ω:-</u>

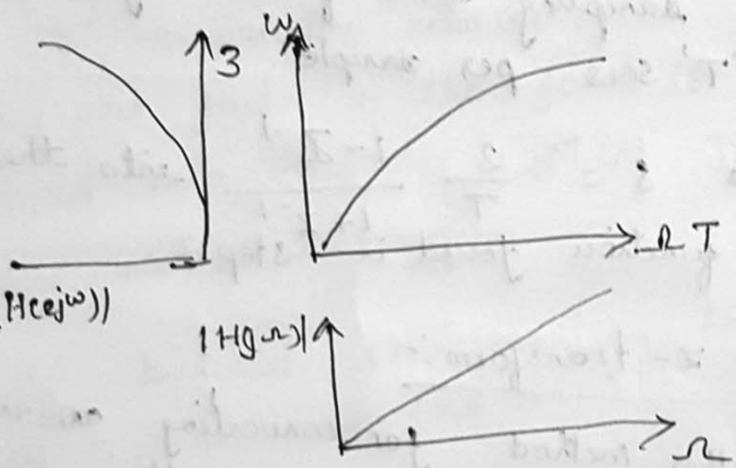


<u>Effect on magnitude response due to Warping effect:</u>



Prewarping: The effect of the non-linear compression at high frequencies can be compensated. When the desired magnitude response is piece-wise constant over frequency, this compression can be compensated by introducing a suitable prescaling, or prewarping the critical frequencies by using the formula

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$



The effect on Phase response due to Warping effect.

∴ we have $\Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$

and $\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$ //

Steps to design digital filter using bilinear transform technique:

1. From the given specifications, find prewarping analog frequencies using formula $\Omega = \frac{2}{T} \tan \frac{\omega}{2}$.
2. Using the analog frequencies find H(S) of the analog filter.
3. Select the sampling rate of the digital filter, call it 'T' secs per sample.
4. Substitute $s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}$ into the transfer function found in step 2.

The matched z-transform:-

Another method for converting an analog filter into an equivalent digital filter is to map the poles and zeros of H(S) directly into poles and zeros in the z-plane. If

$$H(S) = \frac{\prod_{k=1}^{M} (S - z^{k})}{\prod_{k=1}^{N} (S - P_K)}$$

where $\{s_k\}$ → Zeros.

$\{P_k\}$ → Poles of the filter, then the system function of the digital filter is

$$H(z) = \frac{\prod_{k=1}^{M} (1-e^{z_k T} z^{-1})}{\prod_{k=1}^{N} (1-e^{P_k T} z^{-1})}$$

'T' → Sampling interval.

Thus each factor of the form $(s-a)$ in $H(s)$ is mapped into the factor $1-e^{aT}z^{-1}$. This mapping is called matched z-transform.

1) Apply bilinear transformation to $H(s) = \frac{2}{(s+1)(s+2)}$ with $T=1$ sec and find $H(z)$.

Soln: Given $H(s) = \frac{2}{(s+1)(s+2)}$

Substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to get $H(z)$.

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{2}{(s+1)(s+2)}\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

Given $T = 1$ Sec

$$H(z) = \frac{2}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+1\right\}\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+2\right\}}$$

$$= \frac{2(1+z^{-1})^2}{(3-z^{-1})(4)}$$

$$= \frac{(1+z^{-1})^2}{6-2z^{-1}} = \frac{0.166(1+z^{-1})^2}{(1-0.33z^{-1})}$$ //

2) Using the bilinear transform, design a high-pass filter, monotonic in passband with cutoff frequency of 1000 Hz and down 10dB at 350 Hz. The sampling frequency is 5000 Hz.

Given: $\alpha_p = 3dB$; $\omega_c = \omega_p = 2\times\pi\times1000 = 2000\pi$ rad/sec.

$\alpha_s = 10dB$; $\omega_s = 2\times\pi\times350 = 700\pi$ rad/sec.

$T = \frac{1}{f} = \frac{1}{5000} = 2\times10^{-4}$ Sec. //

The characteristics are monotonic in both passband and stop band. ∴ The filter is Butterworth filter.

Prewarping the digital frequencies we have,

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times10^{-4}}\tan\left(\frac{2000\pi\times2\times10^{-4}}{2}\right)$$

$$= 10^4\tan(0.2\pi) = 7265 \text{ rad/sec.}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times10^{-4}}\tan\frac{(700\pi\times2\times10^{-4})}{2}$$

$$= 10^4\tan(0.07\pi) = 2235 \text{ rad/sec.}$$

The order of the filter

$$N = \frac{\log\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\dfrac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\dfrac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\dfrac{7265}{2235}}$$

$$= \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

$$\therefore N = 1$$

The first-order Butterworth filter for $\Omega_c = 1$ rad/sec

is $H(s) = \frac{1}{1+s}$.

The highpass filter for $\Omega_c = \Omega_p = 7265$ rad/sec can be obtained by using the transformation

$$s \rightarrow \frac{\Omega_c}{s}$$

i.e. $s \rightarrow \frac{(7265)}{s}$

The transfer function of high pass filter

$$H(s) = \frac{1}{s+1}\Bigg|_{s = \frac{7265}{s}}$$

$$= \frac{s}{s+7265}$$

Using bilinear transformation

$$H(z) = H(s)\Bigg|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{s}{s+7265}\Bigg|_{s = \frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{1000\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{0.5762(1-z^{-1})}{1-0.1584z^{-1}}$$

Realization of Digital Filters:-

Two types: (1) Recursive
(2) Non-recursive.

1) Recursive: The current output $y(n)$ is a function of past outputs, past and present inputs. This form correspondence to IIR filter.

2) Non-recursive:- The current o/p sample $y(n)$ is a function of only past and present inputs (FIR filters).

IIR filter can be realized in many forms. They are

1. Direct form - I realization
2. Direct form - II "
3. Transposed direct form realization.
4. Cascade form realization
5. Parallel form "
6. Lattice - ladder structure.

Direct form I realization :-

1. Obtain the direct form - I realization for the system described by difference equation

$$y(n) = 0.5\,y(n-1) - 0.25\,y(n-2) + x(n) + 0.4\,x(n-1).$$

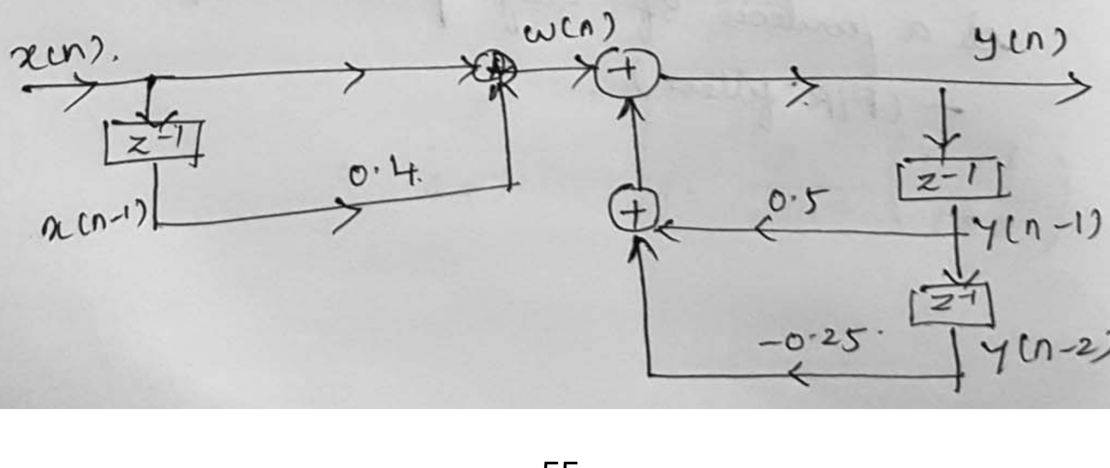
Soln:

Let

$$x(n) + 0.4\,x(n-1) = w(n)$$

then

$$y(n) = 0.5\,y(n-1) - 0.25\,y(n-2) + w(n)$$



Direct form - II:

2) Determine the direct form II realization for the following system $y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$.

Soln: The system function is given by

$$\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

Let $\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2}$

$$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$$

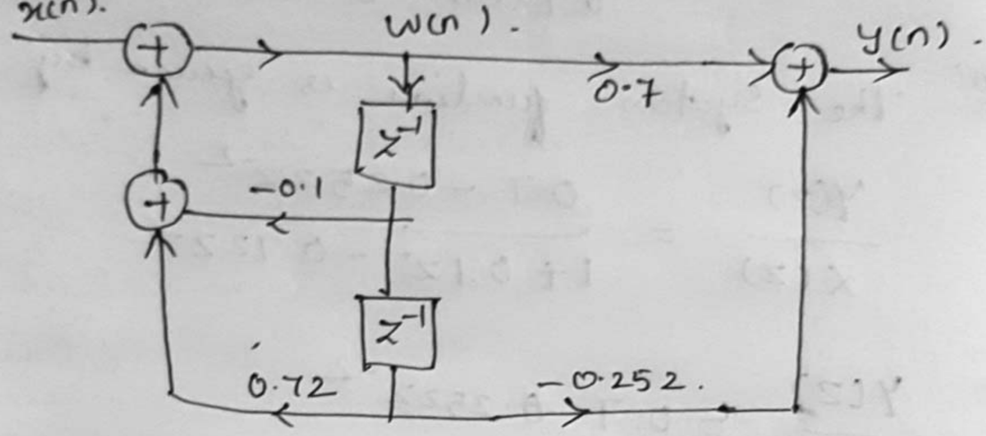
Then $y(n) = 0.7w(n) - 0.252w(n-2)$.

Illy let $\frac{W(z)}{X(z)} = \frac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$

$$W(z) = X(z) - 0.1z^{-1}W(z) + 0.72z^{-2}W(z)$$

then $w(n) = x(n) - 0.1w(n-1) + 0.72w(n-2)$

Direct combine the equations to form the realization of the system.



Transposition theorem and transposed structure.

It is defined by the following operations

(i) Reverse the direction of all branches in the signal flow graph.

(ii) Interchange the i/p's and o/p's

(iii) Reverse the roles of all nodes in the flow graph.

(iv) Summing points become branching points.

(v) Branching points become summing points.

According to transposition theorem, the system transfer function remain unchanged by transposition.

7) Determine the direct form II and Transposed direct form II for the given system

$$y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$$

Soln: The system transfer function of the given difference equation is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-0.5z^{-1}+0.25z^{-2}}$$

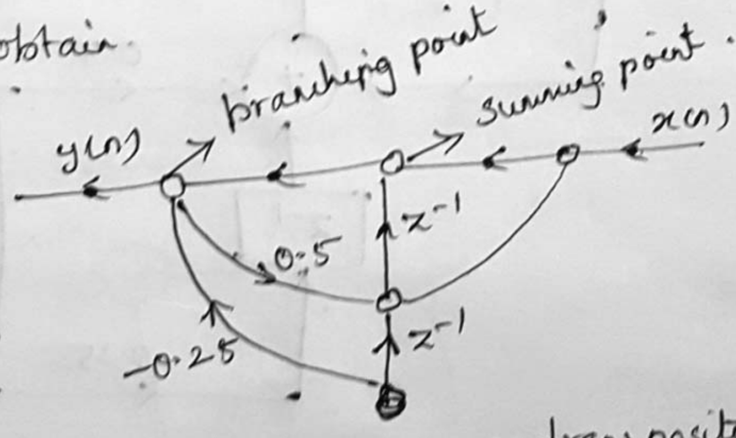
To get transposed direct form II do the following operations

(i) Change the direction of all branches.

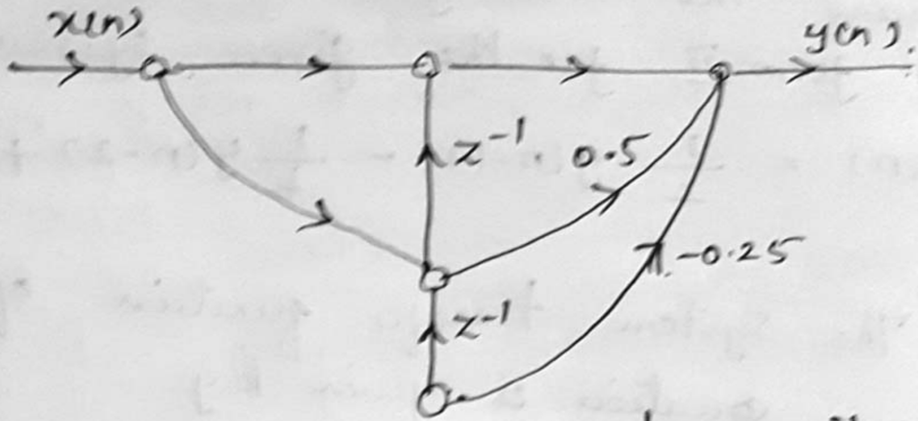
(ii) Interchange the input and output.

(iii) change the summing point to branching point and vice versa.

Then we obtain.

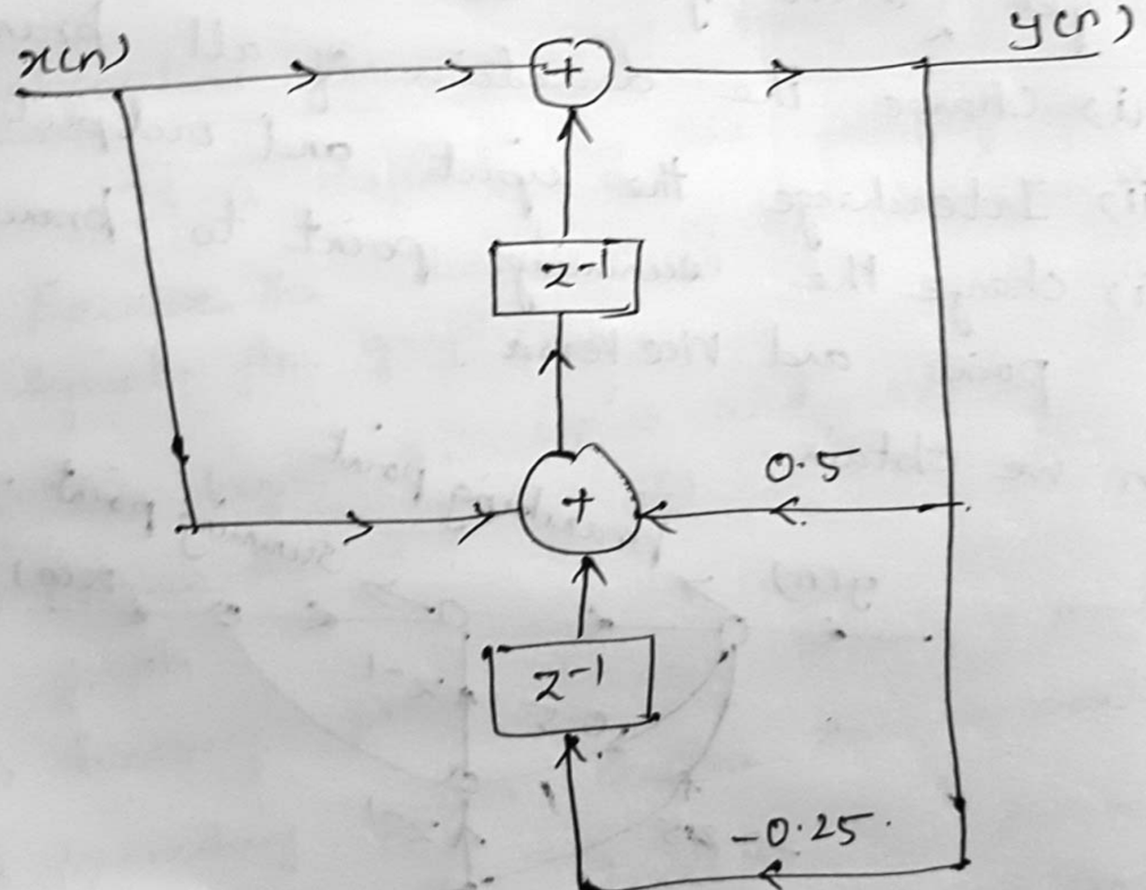


(i) steps of operation in transposition.



(ii) Steps of operation in transposition.

Transposition Structure:-



Cascade form:-

1) Realize the system with difference equation

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{3}x(n-1)$$

in cascade form.

Soln:- From the difference equation,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-3/4z^{-1}+\frac{1}{8}z^{-2}}$$

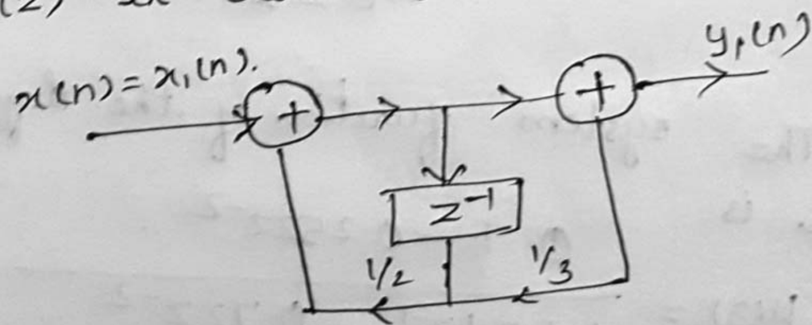
$$= \frac{1+\frac{1}{3}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{4}z^{-1}\right)}$$

$$= H_1(z)H_2(z)$$

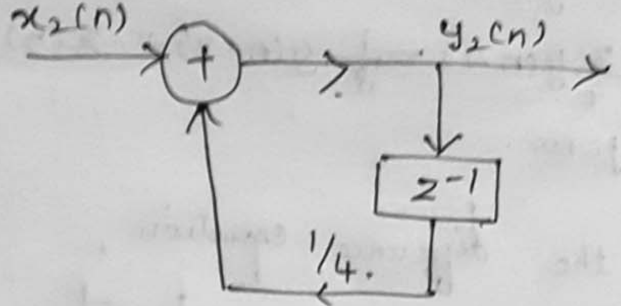
Where

$$H_1(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}} \quad \text{and} \quad H_2(z) = \frac{1}{1-\frac{1}{4}z^{-1}}$$

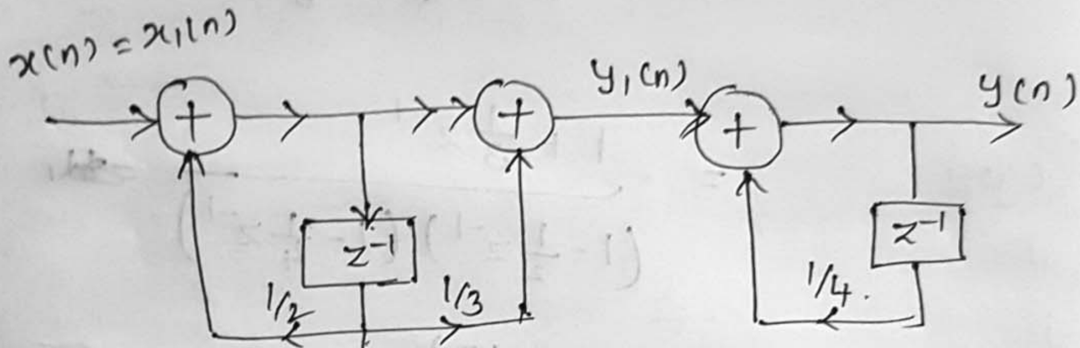
$H_1(z)$ in direct form II as



IIIrly $H_2(z) \Rightarrow$



Cascading the realization $H_1(z)$ & $H_2(z) \Rightarrow$



Parallel form Structure:

1) Realize the system given by difference equation
$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$ in parallel form.

Soln:- The system function of the difference equation is

$$H(z) = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

Soln:- The system function of the difference equation is

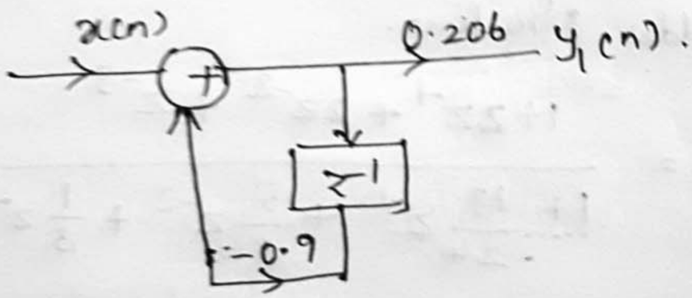
$$H(z) = \frac{0.7 - 0.252z^{-2}}{1+0.1z^{-1}-0.72z^{-2}}$$

$$= 0.35 + \frac{0.35-0.035z^{-1}}{1+0.1z^{-1}-0.72z^{-2}}$$

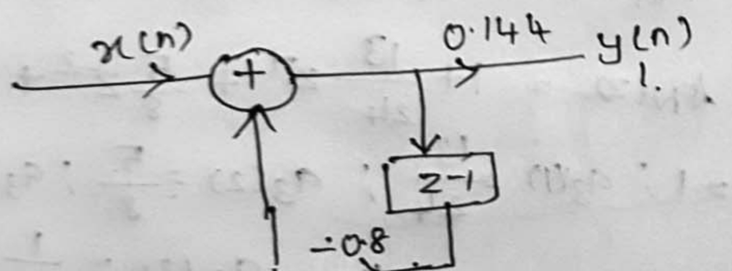
$$= 0.35 + \frac{0.206}{1+0.9z^{-1}} + \frac{0.144}{1-0.8z^{-1}}$$

$$\boxed{H(z) = C + H_1(z) + H_2(z)}$$

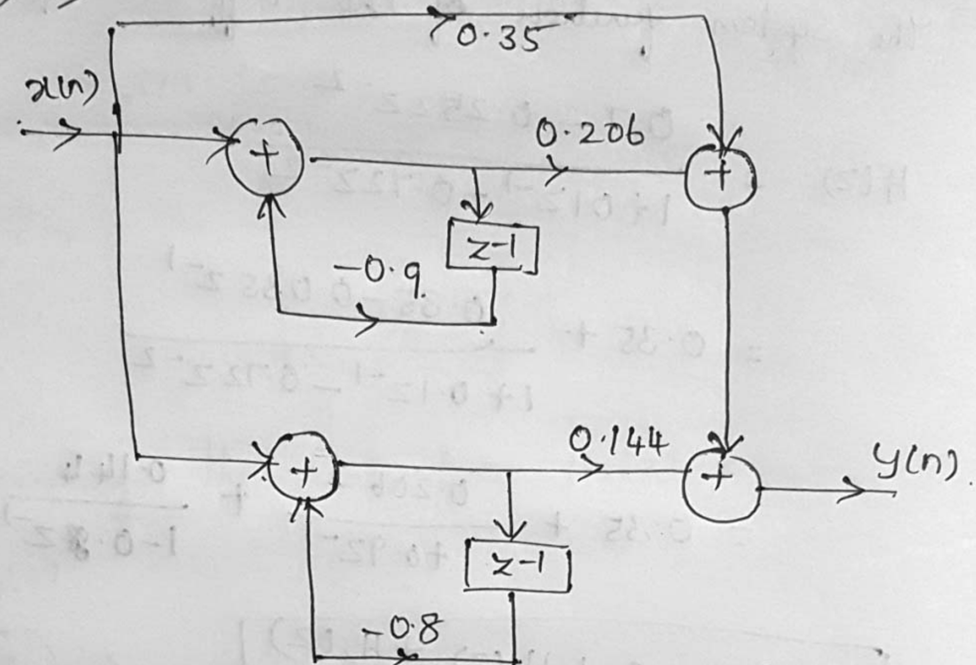
$H_1(z) \Rightarrow$ (direct form II as)



$H_2(z) \Rightarrow$ (direct form II as)



$H(Z) \Rightarrow$



Lattice - Ladder structure:

1) Convert the following pole zero IIR filter into a lattice - ladder structure.

$$H(Z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}}$$

Soln:

Given $b_M(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3}$

$$A_N(z) = 1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}$$

$a_3(0) = 1$; $a_3(1) = \frac{13}{24}$; $a_3(2) = \frac{5}{8}$; $a_3(3) = \frac{1}{3}$.

$$\therefore K_3 = a_3(3) = \frac{1}{3}.$$

We know

$$a_{m-1}(k) = \frac{a_m(k) - a_m(m)\, a_m(m-k)}{1 - a^2_m(m)}$$

For m = 3 and k = 1

$$a_2(1) = \frac{a_3(1) - a_3(3)\, a_3(2)}{1 - a^2_3(3)}$$

$$= \frac{\frac{13}{24} - \frac{1}{3}\left(\frac{5}{8}\right)}{1 - (2/3)^2} = \frac{3}{8}$$

For m = 3 and k = 2

$$k_2 = a_2(2) = \frac{a_3(2) - a_3(3)\, a_3(1)}{1 - a^2_3(3)}$$

$$= \frac{\frac{5}{8} - \frac{1}{3}\left(\frac{13}{24}\right)}{1 - (1/3)^2} = \frac{1}{2}$$

For m = 2 and k = 1

$$k_1 = a_1(1) = \frac{a_2(1) - a_2(2)\, a_2(1)}{1 - a_2^2(2)}$$

$$= \frac{\frac{3}{8} - \frac{1}{2}\left(\frac{3}{8}\right)}{1 - (1/2)^2} = \frac{1}{4}$$

∴ for lattice structure

$$k_1 = \frac{1}{4}\ ;\ k_2 = \frac{1}{2}\ ;\ k_3 = \frac{1}{3}.$$

For Ladder structure.

$$c_m = b_m - \sum_{i=m+1}^{M} c_i a_i(i-m)\ ; \quad m = M, M-1, \cdots, 0.$$

$$c_3 = b_3 = 1.$$

$$c_2 = b_2 - c_3 a_3(1)$$

$$= 2 - 1\left(\frac{13}{24}\right) = 1.4583$$

$$c_1 = b_1 - \sum_{i=2}^{3} c_i a_i(i-m)$$

$$= b_1 - \left[c_2 a_2(1) + c_3 a_3(2)\right]$$

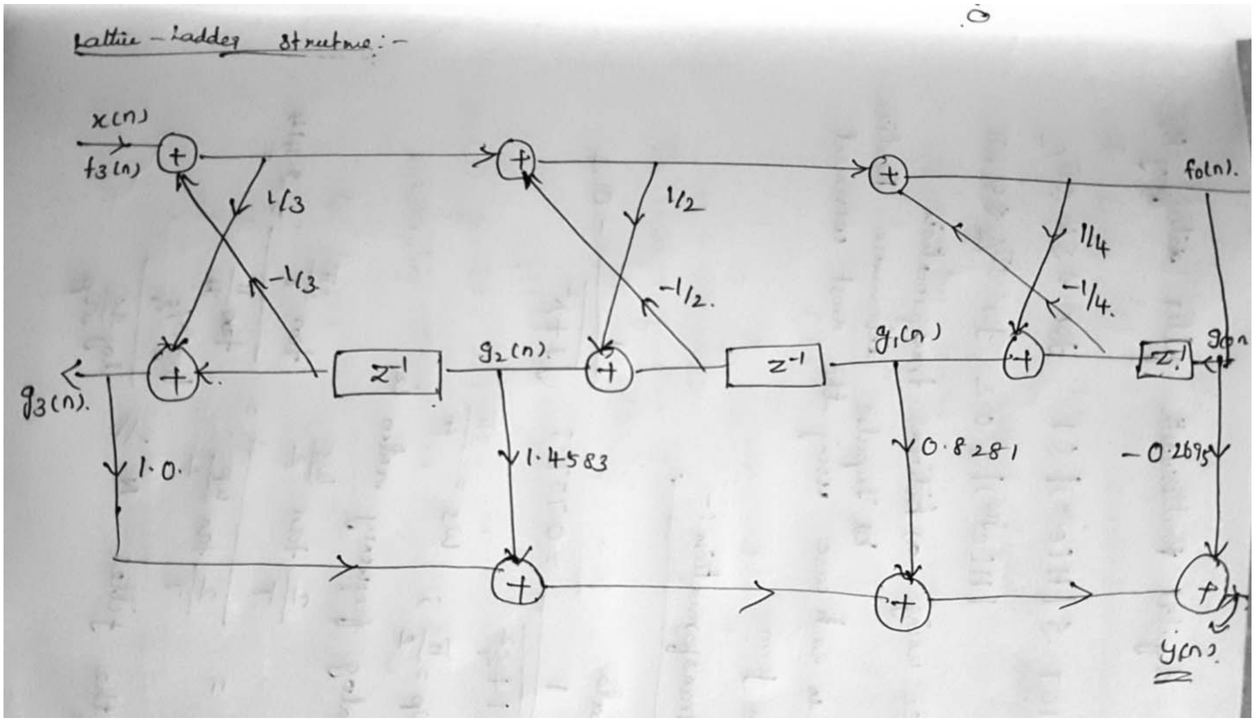
$$= 2 - \left[(1.4583)(3/8) + 5/8\right] = 0.8281.$$

$$c_0 = b_0 - \sum_{i=1}^{3} c_i a_i(i-m).$$

$$= b_0 - \left[c_1 a_1(1) + c_2 a_2(2) + c_3 a_3(3)\right]$$

$$= 1 - \left[0.8281\left(\frac{1}{4}\right) + 1.4583\left(\frac{1}{2}\right) + \frac{1}{3}\right]$$

$$= -0.2695$$


Lattice - Ladder Structure:-
x(n)
f3(n)
1/3
-1/3
1/2
-1/2
1/4
-1/4
f0(n).
z⁻¹
z⁻¹
z⁻¹
g3(n).
g2(n).
g1(n)
1.0.
1.4583
0.8281
-0.2695
y(n).

(X)

1. Design a digital Butterworth filter satisfying the conditions.

$$0.707 \le |H(e^{j\omega})| \le 1 \quad \text{for } 0 \le \omega \le \pi/2$$

$$|H(e^{j\omega})| \le 0.2 \quad \text{for } 3\pi/4 \le \omega \le \pi.$$

with T = 1 Sec using a) bilinear transformation
b) Impulse invariance. Realize the filter in each case using the most convenient realization form.

a) Bilinear transformation :-

Given data

$$\frac{1}{\sqrt{1+\varepsilon_1^2}} = 0.707 \; ; \quad \frac{1}{\sqrt{1+\lambda^2}} = 0.2.$$

$$\omega_p = \frac{\pi}{2} \; ; \quad \omega_s = \frac{3\pi}{4}$$

The analog frequency ratio is

$$\frac{\Omega_s}{\Omega_p} = \frac{\frac{2}{T}\tan\frac{\omega_s}{2}}{\frac{2}{T}\tan\frac{\omega_p}{2}} = \frac{\tan\frac{3\pi}{8}}{\tan\frac{\pi}{4}} = 2.414$$

Order of the filter. $N \ge \dfrac{\log \lambda/\varepsilon_1}{\log \frac{\Omega_s}{\Omega_p}}$

From the given data $\lambda = 4.898$ $\varepsilon = 1$

So $N \geq \frac{\log 4.898}{\log 2.414} = 1.803$.

Rounding N to nearest highest integer value we get $N = 2$. We know

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \Omega_p \quad (\because \varepsilon = 1)$$

$$= \frac{2}{T} \tan\frac{\omega_p}{2} = 2\tan\frac{\pi}{4} = 2 \text{ rad/sec}$$

The transfer function of second order normalized Butterworth filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}.$$

$H_a(s)$ for $\Omega_c = 2$ rad/sec can be obtained by substituting $s \rightarrow s/2$ in $H(s)$.

i.e. $$H_a(s) = \frac{1}{(s/2)^2 + \sqrt{2}\cdot(s/2) + 1}$$

$$= \frac{4}{s^2 + 2.828s + 4}$$

By using bilinear transformation $H(z)$ can be obtained as

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

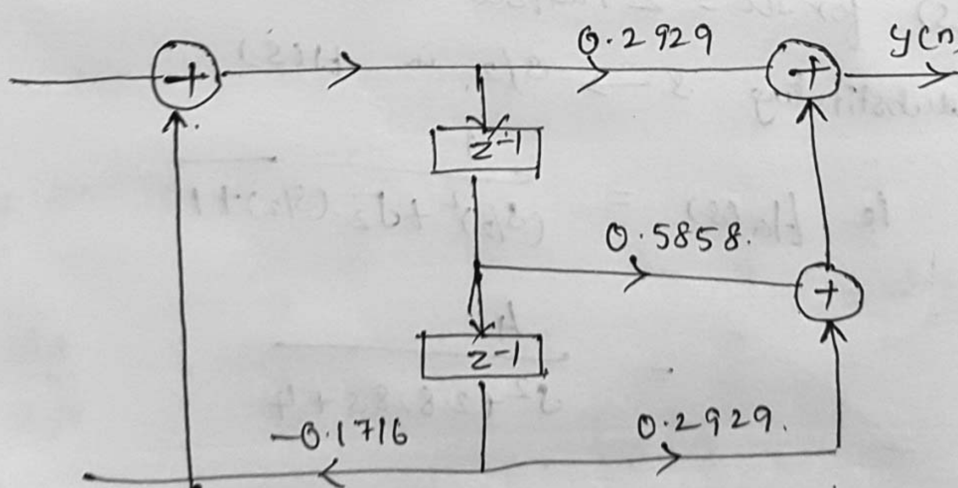
Thus

$$H(z) = \frac{4}{s^2+2.828s+4}\Bigg|_{s=2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} \quad (\because T = 1\,\text{Sec})$$

$$= \frac{4(1+z^{-1})^2}{4(1-z^{-1})^2 + 5.656(1-z^{-2}) + 4(1+z^{-1})^2}$$

$$= \frac{0.2929(1+z^{-1})^2}{1+0.1716z^{-2}}$$

$H(z) \Rightarrow$



b) Impulse Invariant Method:-

Soln: The relationship between analog & digital frequencies in Impulse invariant method is $\omega = \Omega T$.

From the given data $T = 1$ sec i.e. $\omega = \Omega$

$\Rightarrow \Omega_p = \omega_p$; $\Omega_s = \omega_s$

W.k. $\lambda = 4.898$; $\varepsilon = 1$

The order of the filter

$$N \geq \frac{\log \lambda/\varepsilon}{\log \frac{\Omega_s}{\Omega_p}} = \frac{\log 4.989}{\log \frac{3\pi/4}{\pi/2}}$$

$$\boxed{N \geq 3.924}$$

i.e. $N = 4$

The transfer function of a fourth order normalized Butterworth filter is

$$H(s) = \frac{1}{(s^2+0.76537+1)(s^2+1.84775s+1)}$$

As $\varepsilon = 1$; $\Omega_p = \Omega_c = 0.5\pi = 1.57$.

$$H_a(s) = H(s)\Big|_{s \to \frac{s}{1.57}}$$

$$= \frac{(1.57)^4}{(s^2+1.202s+2.465)(s^2+2.902s+2.465)}$$

Ha(s) in the partial fraction form is given by

$$Ha(s) = \frac{A}{(s+1.45+j0.6)} + \frac{A^*}{(s+1.45-j0.6)}$$

$$+ \frac{B}{(s+0.6+j1.45)} + \frac{B^*}{(s+0.6-j1.45)}$$

$$A = (s+1.45+j0.6) \left. \frac{(1.57)^4}{(s+1.45+j0.6)(s+1.45-j0.6)(s^2+1.202s+2.465)} \right|_{s=-1.45-j0.6}$$

$$= \frac{(1.57)^4}{(-j0.6-0.6)\left[(-1.45-j0.6)^2+1.202(-1.45-j0.6)+2.465\right]}$$

$$= \frac{(1.57)^4}{-j(1.2)\left[1.7425+1.74j-1.7429-j0.7212+2.465\right]}$$

$$= \frac{(1.57)^4}{-j(1.2)\,(2.465 + j\,1.0188)}$$

$$= \frac{5.063}{1.0188 - j\,2.465} = \frac{5.063\,(1.0188 + j2.465)}{7.114}$$

$$= 0.7116\,(1.0188 + j\,2.465) = 0.7253 + j\,1.754$$

$$B = (s + 0.6 + j\,1.45)\,\frac{(1.57)^4}{(s + 0.6 + j\,1.45)\,(s + 0.6 - j\,1.45)\,(s^2 + 2.902s + 2.465)}\Bigg|_{s = -0.6 - j1.45}$$

$$= \frac{(1.57)^4}{-j(2.9)\left[(-0.6 - j1.45)^2 + 2.902(-0.6 - j1.45) + 2.465\right]}$$

$$= \frac{(1.57)^4}{-j(2.9)\left[-1.7425 + j1.74 - 1.7412 - j\,4.208 + 2.465\right]}$$

$$= \frac{2.095}{-j\,(-1.0187 - j2.468)}$$

$$= \frac{2.095}{-2.469 + j1.0187}$$

$$= \frac{2.095[-2.468 - j1.0187]}{7.1287}$$

$$= 0.29388[-2.468 - j1.0187]$$

$$= -0.7253 - 0.3j$$

$$H_a(s) = \frac{0.7253 + j1.754}{s-(-1.45-j0.6)} + \frac{0.7253 - j1.754}{s-(-1.45+j0.6)}$$

$$+ \frac{-0.7253 - 0.3j}{s-(-0.6-j1.45)} + \frac{-0.7253 + 0.3j}{s-(-0.6+j1.45)}$$

W.k. T = 1 sec

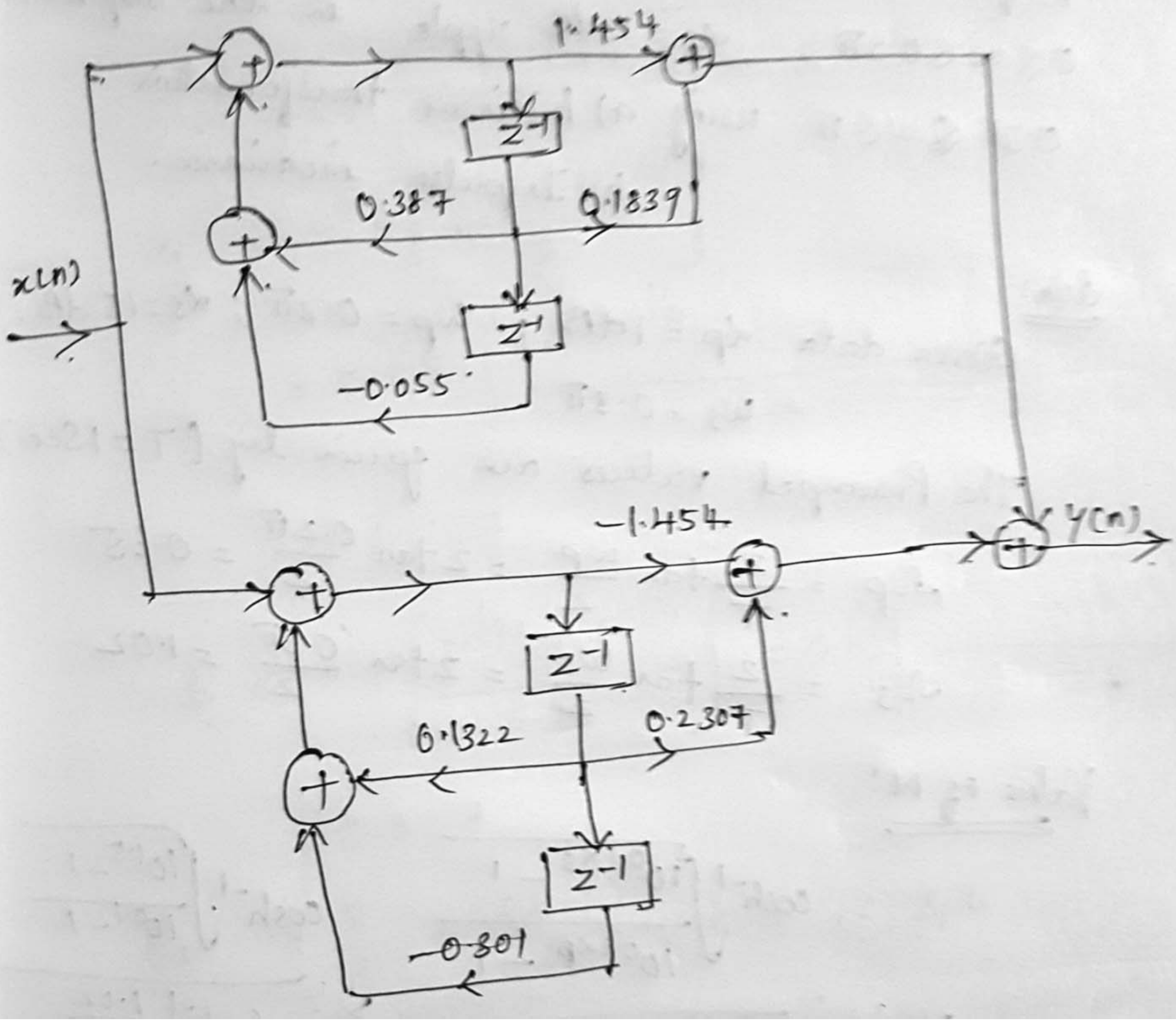
$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1-e^{p_k}z^{-1}}$$

$$\therefore H_a(s) = \frac{0.7253 + j1.754}{1-e^{-1.45}e^{-j0.6}z^{-1}} + \frac{0.7253 - j1.754}{1-e^{-1.45}e^{j0.6}z^{-1}}$$

$$+ \frac{-(0.7253 + 0.3j)}{1-e^{-0.6}e^{-j1.45}z^{-1}} + \frac{-0.7253 + 0.3j}{1-e^{-0.6}e^{j1.45}z^{-1}}$$

$$= \frac{1.454 + 0.1839z^{-1}}{1-0.387z^{-1}+0.055z^{-2}} + \frac{-1.454 + 0.2307z^{-1}}{1-0.1322z^{-1}+0.301z^{-2}}$$

This can be realized ⇒



2) Design a Chebyshev lowpass filter with the specifications $\alpha_p = 1dB$ ripple in the passband $0 \leq \omega \leq 0.2\pi$, $\alpha_s = 15dB$ ripple in the stop band $0.3\pi \leq \omega \leq \pi$ using a) bilinear transformation. b) Impulse invariance.

Soln:

Given data $\alpha_p = 1dB$; $\omega_p = 0.2\pi$; $\alpha_s = 15\,dB$.

$\omega_s = 0.3\pi$.

The Prewarped values are given by $(T = 1\,Sec)$.

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2} = 2\tan \frac{0.2\pi}{2} = 0.65$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2} = 2\tan \frac{0.3\pi}{2} = 1.02$$

Value of N:

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1} \dfrac{\Omega_s}{\Omega_p}} \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.5} - 1}{10^{0.1} - 1}}}{\cosh^{-1} \dfrac{1.02}{0.65}}$$

$= 3.01$

Let us take $N = 4$

Axis of the ellipse :-

We know $\varepsilon = \sqrt{10^{0.1\alpha_p} - 1} = 0.508$.

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 4.17.$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right]$$

$$= 0.65 \left[\frac{(4.17)^{1/4} - (4.17)^{-1/4}}{2} \right]$$

$$= 0.237.$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right] = 0.65 \left[\frac{(4.17)^{1/4} + (4.17)^{-1/4}}{2} \right]$$

$$= 0.6918$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad k = 1, 2, 3, 4.$$

$\phi_1 = 112.5^\circ$; $\phi_2 = 157.5^\circ$; $\phi_3 = 202.5^\circ$

$\phi_4 = 247.5^\circ$.

The poles are:

$$S_k = a\cos\phi_k + jb\sin\phi_k \quad ; \; k = 1, 2, 3, 4.$$

$$S_1 = a\cos\phi_1 + jb\sin\phi_1 = 0.237\cos 112.5^\circ + j0.6918\sin 112.5^\circ$$

$$= -0.0907 + j0.639.$$

$$S_2 = a\cos\phi_2 + jb\sin\phi_2$$

$$= 0.237\cos 157.5^\circ + j0.6918\sin 157.5^\circ$$

$$= -0.2189 + j0.2647.$$

$$S_3 = a\cos\phi_3 + jb\sin\phi_3.$$

$$= 0.237\cos 202.5^\circ + j0.6918\sin 202.5^\circ$$

$$= -0.2189 - j0.2647.$$

$$S_4 = a\cos\phi_4 + jb\sin\phi_4$$

$$= 0.237\cos 247.5^\circ + j0.6918\sin 247.5^\circ$$

$$= -0.0907 - j0.639.$$

The denominator polynomial of

$$H(s) = \left[(s+0.0907)^2 + (0.639)^2\right]\left[(s+0.2189)^2 + (0.2647)^2\right]$$

$$= (s^2 + 0.1814s + 0.4165)(s^2 + 0.4378s + 0.118)$$

As 'N' is even, the numerator of $H(s)$

$$H(s) = \frac{(0.4165)(0.118)}{\sqrt{1+\epsilon^2}} = 0.04381.$$

The transfer function

$$H(s) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.1180)}$$

The z-transform of the digital filter

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.1180)}\Bigg|_{s = 2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]} \quad \left[\because T = 1\sec\right]$$

$$= \frac{0.04381(1+z^{-1})^4}{\left\{4(1-z^{-1})^2 + 0.1814 \times 2(1-z^{-2}) + 0.4165(1+z^{-1})^2\right\} \left\{4(1z^{-1})^2 + 0.4378 \times 2(1-z^{-2}) + 0.1180(1+z^{-1})^2\right\}}$$

$$= \frac{0.04381(1+z^{-1})^4}{(4.7794 - 7.1668z^{-1} + 4.0538z^{-2})(4.9936 - 7.7642z^{-1} + 3.2424z^{-2})}$$

$$= \frac{0.001836(1+z^{-1})^4}{(1-1.499z^{-1} + 0.8482z^{-2})(1-1.5548z^{-1} + 0.6493z^{-2})}$$

b) <u>Impulse Invariance method:-</u>

Given data $\omega_p = 0.2\pi$; $\omega_s = 0.3\pi$; $\alpha_p = 1dB$;
$\alpha_s = 15dB$.

Analog frequency ratio $\frac{\Omega_s}{\Omega_p} = \frac{\omega_s}{\omega_p} = \frac{0.3\pi}{0.2\pi} = 1.5$.

($\because \omega = \Omega T$ and $T = 1sec$)

Value of N =>

$$N \geq \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\frac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1}\sqrt{\frac{10^{1.5}-1}{10^{0.1}-1}}}{\cosh^{-1}(0.5)}$$

$= 3.2$

we get $\boxed{n = 4}$

Axis of ellipse :-

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \quad ; \quad k = 1, 2, \ldots, N.$$

$$\phi_1 = 112.5^\circ \; ; \; \phi_2 = 157.5^\circ \; ; \; \phi_3 = 202.5^\circ$$

$$\phi_4 = 247.5^\circ.$$

$$\varepsilon = \sqrt{10^{0.1\alpha p} - 1} = \sqrt{10^{0.1} - 1} = 0.508$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 4.17.$$

$$a = \Omega p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right]$$

$$= 0.2\pi \left[\frac{4.17^{1/4} - 4.17^{-1/4}}{2} \right] = 0.229.$$

$$b = \Omega p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right]$$

$$= 0.2\pi \left[\frac{4.17^{1/4} + 4.17^{-1/4}}{2} \right] = 0.67$$

The poles of the filter

$$s_k = a\cos\phi_k + jb\sin\phi_k \; ; \; k = 1, 2, \cdots, 4$$

$$s_1 = a\cos\phi_1 + jb\sin\phi_k \; ; \; k = 1, 2, \cdots, 4$$

$$= -0.0876 + j0.619$$

$$s_2 = a\cos\phi_2 + jb\sin\phi_2 = -0.2115 + j0.2564$$

$$s_3 = a\cos\phi_3 + jb\sin\phi_3 = -0.2115 - j0.2564$$

$$s_4 = a\cos\phi_4 + jb\sin\phi_4 = -0.0876 - j0.619.$$

The denominator polynomials of

$$H(s) = \{(s+0.0876)^2 + (0.619)^2\}\{(s+0.215)^2 + (0.2564)^2\}$$

$$= (s^2 + 0.175s + 0.391)(s^2 + 0.423s + 0.11)$$

$$= \frac{A}{s-(-0.0876+j0.619)} + \frac{A^*}{s-(-0.0876-j0.619)}$$

$$+ \frac{B}{s(-(-0.2115+j0.2564))} + \frac{B^*}{s-(-0.2115-j0.2564)}$$

Solving for A A^*, B, B^* using

$$A = -0.0413 + j0.0814$$

$$B = 0.0413 - j0.2166$$

Impulse Invariant transform

i.e. $\sum_{K=1}^{N} \frac{C_k}{s - P_K} = \sum_{K=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$

We obtain

$$H(z) = \frac{-0.083 - 0.0245z^{-1}}{1 - 1.49z^{-1} + 0.839z^{2}} + \frac{0.083 + 0.0238z^{-1}}{1 - 1.56z^{-1} + 0.655z^{-2}}$$ //

4. Design a Butterworth filter using the impulse variance method for the following specifications.

$$0.8 \leq |H(e^{j\omega})| \leq 1 \qquad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.2 \qquad 0.6\pi \leq \omega \leq \pi.$$

Soln:

Given $\frac{1}{\sqrt{1+\xi^2}} = 0.8$ from which $\xi = 0.75$,

$\frac{1}{\sqrt{1+\xi^2}} = 0.2$ from which $\lambda = 4.809$.

$\omega_s = 0.6\pi$ rad ; $\omega_p = 0.2\pi$ rad

$$\frac{\omega_s}{\omega_p} = \frac{\Omega_s T}{\Omega_p T} = \frac{\Omega_s}{\Omega_p} = \frac{0.6\pi}{0.2\pi} = 3.$$

$$N = \frac{\log \lambda/\varepsilon}{\log 1/k} = \frac{\log \frac{4.809}{0.75}}{\log 3} = 1.71.$$

$\therefore$ $\boxed{N = 2.}$

For $N^* = 2$ the transfer function of normalized Butterworth filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}} = \frac{0.2\pi}{(0.75)^{1/2}} = 0.231\pi.$$

$$H_a(s) = H(s)\Big|_{s \to s/0.231\pi}$$

$$= \frac{0.5266}{s^2 + 1.033s + 0.5266}$$

$$= \frac{0.156j}{s + 0.51 + j0.51} - \frac{0.516j}{s + 0.51 - j0.51}$$

$$= \frac{0.516j}{s-(-0.51-j0.51)} - \frac{0.516j}{s-(-0.51+j0.51)}$$

$$H(z) = \frac{0.516j}{1-e^{-0.51T}e^{-j0.51T}z^{-1}} - \frac{0.516j}{1-e^{-0.51T}e^{j0.51T}z^{-1}}$$

$\because T = 1\,sec$

$$H(z) = \frac{0.3019z^{-1}}{1-1.048z^{-1}+0.36z^{-2}}.$$

Finite Impulse Response Filters:

Advantages:-

1. Always stable
2. FIR filters with exactly linear phase can be easily designed.
3. can be realized in both recursive and non-recursive structures.
4. free of limit cycle oscillations, when implemented on a finite word length digital system.
5. Excellent design methods are available for various kinds of FIR filters.

Disadvantages:-

1) Very costly.
2) requires more arithmetic operations & hardware components (multipliers, adders and delay elements).
3) Memory requirement and execution time are very high.

Distinguish between FIR and IIR filters.

S.NO	FIR Filter	IIR filter.
1.	can be easily designed to have perfectly linear phase.	do not have linear phase.
2.	can be realized recursively and non-recursively.	easily realized recursively
3.	Greater flexibility to control the shape of their magnitude response.	less flexibility, limited to specific kind of filters.
4.	Errors due to roundoff noise are less	The roundoff noise in IIR filters are more.

Different techniques of designing FIR filters:

Three well known methods: for designing FIR filter with linear phase:-

1) Windows method
2) Frequency sampling method.
3) Optimal or minimax design.

Characteristics of linear phase FIR filters-

Four types:-

1) Symmetrical impulse response N even.
2) Symmetrical impulse response N odd.
3) Antisymmetrical impulse response N odd.
4) Antisymmetrical impulse response N even.

Fourier series method of designing FIR filters:-

The frequency response $H(e^{j\omega})$ of any digital filter is periodic in frequency, and can be expanded in a Fourier series

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}.$$

Where the fourier coefficients are

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

h(n) is finite duration, hence the filter resulting from a Fourier series representation of $H(e^{j\omega})$ is an unrealizable FIR filter. To get an FIR filter that approximates $H(e^{j\omega})$ would lie to truncate the infinite Fourier series at

$$N = \pm\left(\frac{N-1}{2}\right)$$

Designing methods:-

1. For the desired frequency response $H_d(e^{j\omega})$, find closed form expression for $h(n)$ using the equation

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega$$

2. Truncate $h_d(n)$ at $n = \pm\left(\frac{N-1}{2}\right)$ to get the finite duration sequence $h(n)$.

3. Find $H(Z)$ using the equation

$$H(Z) = Z^{-(N-1)/2}\left[h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left(Z^{n} + Z^{-n}\right)\right]$$

DisAdvantages of Fourier series method:

1. Infinite duration impulse response is truncated at $n = \pm\left(\frac{N-1}{2}\right)$.
2. Direct truncation will lead to fixed percentage overshoots and undershoots.
3. approximated discontinuity in the frequency response.

Gibb's phenomena (or Gibb's oscillations:-

One possible way of finding an FIR filter that approximates $H(e^{j\omega})$ would be to truncate the infinite Fourier series at $n = \pm\left(\frac{N-1}{2}\right)$. Abrupt truncation of the series will lead to oscillation both in passband and in stop band. This phenomenon is known as Gibb's phenomenon.

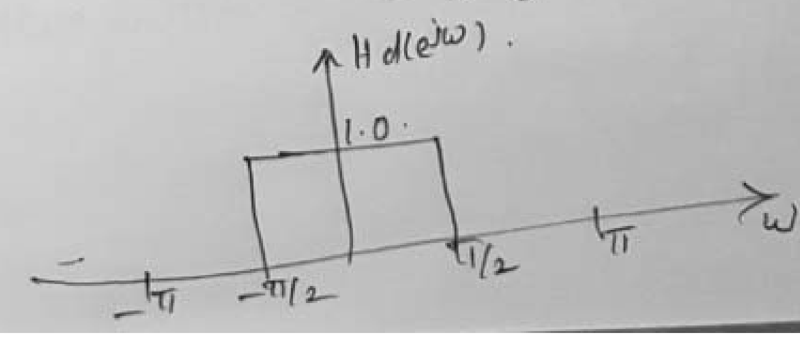
1. Design an ideal lowpass filter with a frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } -\pi/2 \leq \omega \leq \pi/2$$
$$= 0 \text{ for } \pi/2 \leq |\omega| \leq \pi$$

Find the variables of $h(n)$ for $N=11$. Find $H(z)$. Plot the magnitude response.

Soln: The frequency response of lowpass filter with $\omega_c = \pi/2$ is shown in fig.

Given $H_d(e^{j\omega}) = 1$ for $-\pi/2 \leq \omega \leq \pi/2$
$= 0$ for $\pi/2 \leq |\omega| \leq \pi$.



We know

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega .$$

$$= \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega .$$

$$= \frac{1}{2\pi j n} e^{j\omega n} \Big]_{-\pi/2}^{\pi/2} .$$

$$= \frac{1}{\pi n (2j)} \left[e^{j\pi n/2} - e^{-j\pi n/2} \right]$$

$$= \frac{\sin \frac{\pi}{2} n}{\pi n} \quad -\infty \leq n \leq \infty .$$

Truncating $h_d(n)$ to 11 Samples. we have

$$h(n) = \frac{\sin \frac{\pi}{2} n}{n\pi} \quad \text{for } |n| \leq 5$$

$$= 0 \quad \text{otherwise} .$$

For $n=0$; So

$$h(0) = \lim_{n \to 0} \frac{\sin \pi/2\, n}{\pi n} = \frac{1}{2} \lim_{n \to 0} \frac{\sin \frac{\pi}{2} n}{n\pi/2}$$

$$= 1/2 \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \right]$$

(Or) Substitute $n=0 \Rightarrow$.

$$h(0) = hd(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1\,(d\omega) = \frac{1}{2\pi} \omega \Big|_{-\pi/2}^{\pi/2} = 1/2$$

For $n=1$

$$h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183$$

$$n=2 \Rightarrow h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0.$$

$$n=3 \Rightarrow h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = \frac{-1}{3\pi} = -0.106.$$

$$n=4 \Rightarrow h(4) = h(-4) = \frac{\sin 4\pi/2}{4\pi} = 0$$

$$n=5 \Rightarrow h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.06366$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n) \left(z^{n} + z^{-n} \right) \right]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)\,(z^{n} + z^{-n}).$$

$$= 0.5 + 0.3183\,(z^{1} + z^{-1}) - 0.106\,(z^{3} + z^{-3})$$

$$+ 0.06366\,(z^{5} + z^{-5}).$$

The transfer function of the realizable filter is

$$H'(z) = z^{-(N-1)/2} H(z).$$

$$= z^{-5}\left[0.5 + 0.3183\,(z + z^{-1}) - 0.106\,(z^{3} + z^{-2}) + 0.06366\,(z^{5} + z^{-5})\right]$$

$$= 0.06366 - 0.106 z^{-2} + 0.3183 z^{-4} + 0.5 z^{-5}$$

$$+ 0.3183 z^{-6} - 0.106 z^{-8} + 0.06366 z^{-10}.$$

From the above we have

$h(0) = h(10) = 0.0636$

$h(1) = h(9) = 0$

$h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$

$h(4) = h(6) = 0.3183$

$h(5) = 0.5$

The frequency response is given by

$$\overline{H(e^{j\omega})} = \sum_{n=0}^{5} a_n \cos \omega n \quad \text{where}$$

$$a(0) = h\left[\frac{N-1}{2}\right] = h(5) = 0.5$$

$$a(n) = 2h\left[\frac{N-1}{2} - n\right]$$

$$a(1) = 2h(5-1) = 2h(4) = 0.6366$$

$$a(2) = 2h(5-2) = 2h(3) = 0.$$

$$a(3) = 2h(5-3) = 2h(2) = -0.212$$

$$a(4) = 2h(5-4) = 2h(1) = 0.$$

$$a(5) = 2h(5-5) = 2h(0) = 0.127.$$

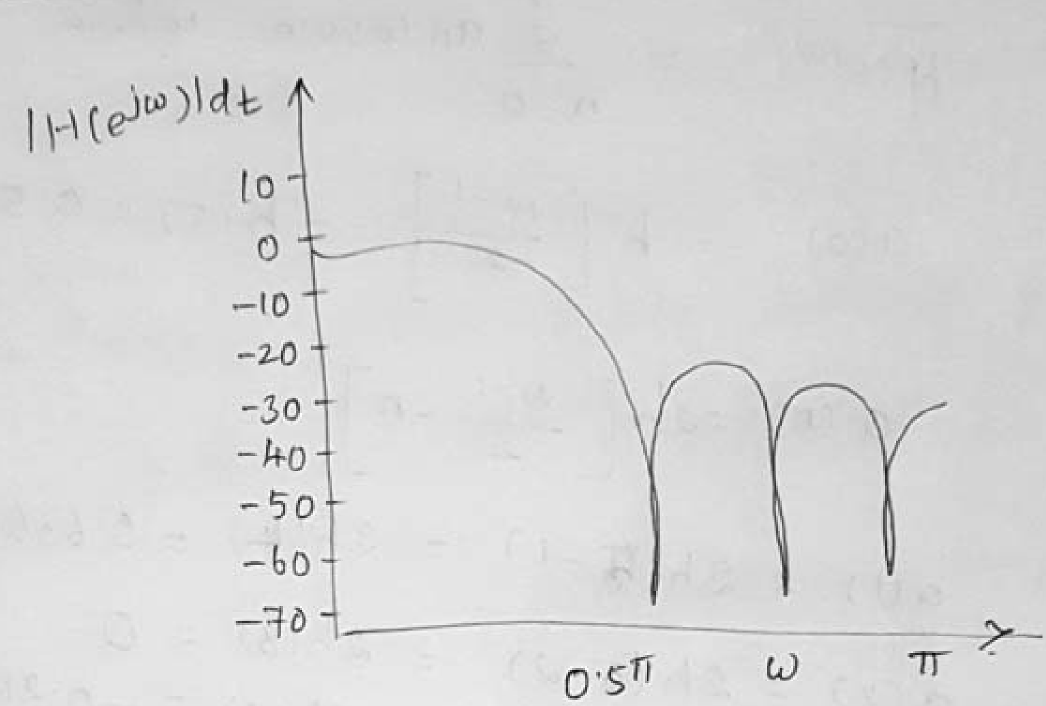
$$\overline{H(e^{j\omega})} = 0.5 + 0.6366 \cos \omega - 0.212 \cos 3\omega + 0.127 \cos 5\omega.$$

Magnitude :

ω (in degrees)	0	10	20	30	40	50	60	70	80
$\lvert H(e^{j\omega}) \rvert_{dB}$	0.4	0.21	-0.26	-0.517	-0.21	0.42	0.77	0.21	-1.79

90	100	110	120	130	140	150	160	170	180.
-6	-14.56	-31.86	-20.6	-26	-32	~~-24~~ 7	~~-30~~ .55	~~-34~~ 32	~~-30.55~~ -26.

The frequency response plot:-



H.W: 1) Design an ideal highpass filter with a frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } \pi/4 \le |\omega| \le \pi$$
$$= 0 \text{ for } |\omega| \le \pi/4$$

Find the values of $h(n)$ for $N = 11$. Find $H(z)$.

Plot the magnitude response.

2) Design an ideal bandreject filter with a desired frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } |\omega| \le \pi/3 \text{ and } |\omega| \ge 2\pi/3$$
$$= 0 \text{ otherwise.}$$

Design of FIR filters using Windows:

steps/ For steps:

Need for employing window technique:-

One possible way of finding an FIR filter that approximates $H(e^{j\omega})$ would be to truncate the infinite Fourier series at $n = \pm(\frac{N-1}{2})$. Aprupt truncation of the series will lead to oscillations in the pass band and stop band. These oscillations can be reduced through the use of less aprupt truncation of the Fourier series. This can be achieved by multiplying the infinite impulse response by a finite weighing sequence $w(n)$, called a window.

Principle of designing windows:-

One possible way of obtaining FIR filter is to truncate the infinite Fourier series at $n = \pm(\frac{N-1}{2})$ where N is the length of the desired sequence. But abrupt truncation of the Fourier series results in oscillation in the passband and stop band. These oscillations are due to slow convergence of the Fourier series. To reduce these oscillations

The Fourier coefficients of the filter are modified by multiplying the infinite impulse response by a finite weighing sequence $w(n)$ called a window, where

$$w(n) = w(-n) \neq 0 \quad \text{for } |n| \leq \frac{N-1}{2}$$
$$= 0 \quad \text{for } |n| > \frac{N-1}{2}$$

After multiplying window sequence $w(n)$ by $h_d(n)$, we get a finite duration sequence $h(n)$ that satisfies the desired magnitude response.

$$h(n) = h_d(n)\, w(n) \quad \text{for all } |n| \leq \frac{N-1}{2}$$
$$= 0 \quad \text{for } |n| > \frac{N-1}{2}.$$

Desirable characteristics of Window :-

1) The central lobe of the frequency response of the window should contain most of the energy and should be narrow.
2) The highest side lobe level of the frequency response should be small.
3) The side lobes of the frequency response should decrease in energy rapidly as ω tends to π.

Procedure for designing FIR filters using windows.

1) For the desired frequency response $H_d(e^{j\omega})$, find the impulse response $h_d(n)$ using Equation

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n}\, d\omega.$$

2) Multiply the infinite impulse response with a chosen window sequence $w(n)$ of length N to obtain filter coefficients $h(n)$ i.e.

$$h(n) = h_d(n)\, w(n) \quad \text{for } |n| \leq \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise}.$$

3) Find the transfer function of the realizable filter.

$$H(z) = z^{-(N-1)/2}\left[h(0) + \sum_{n=0}^{(N-1)/2} h(n)(z^{n}+z^{-n})\right]$$

Types :-

1) Rectangular window.
2) Triangular or Barlett window.
3) Raised cosine window.
4) Hanning window.
5) Hamming window
6) Blackman window

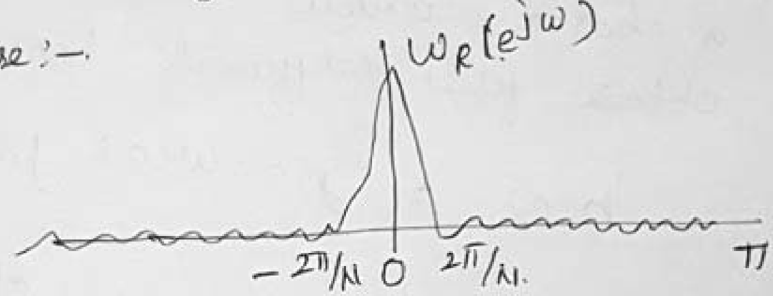
1. Frequency response of N-point rectangular window:-

The frequency response of the rectangular window is given by

$$W_R(e^{j\omega}) = \frac{\sin \frac{\omega N}{2}}{\sin \omega/2}$$

Where $N \rightarrow$ Number of samples.

frequency response:-



frequency response of a Hanning window:-

The frequency response of Raised cosine window is given by

$$W(e^{j\omega}) = \alpha \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}} + \frac{(1-\alpha)}{2} \frac{\sin(\omega N/2 - N\pi/N-1)}{\sin(\omega/2 - \pi/N-1)}$$

$$+ \left(\frac{1-\alpha}{2}\right) \frac{\sin(\omega N/2 + N\pi/N-1)}{\sin(\omega/2 + \pi/N-1)}$$

$\alpha = 0.5$ for Hanning Window

$\alpha = 0.54$ for Hamming Window.

Equation for Hanning window is given by

$$W_{Hn}(n) = 0.5 + 0.5 \cos \frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \leq n \leq (N-1)/2$$

$$= 0 \quad \text{otherwise.}$$

Hamming Window :-

The equation for Hamming window is given by

$$w_H(n) = 0.54 + 0.46 \cos \frac{2\pi n}{N-1} \quad \text{for } -\left(\frac{N-1}{2}\right) \leq n \leq \left(\frac{N-1}{2}\right)$$

$$= 0 \quad \text{otherwise.}$$

Frequency response for Barlett window :-

$$W_T(e^{jw}) = \left[\frac{\sin(w(N-1/4))}{\sin w/2}\right]^2$$

Equation for Barlett window :-

$$\omega_T(n) = 1 - \frac{2|n|}{N-1} \quad \text{for } -(N-1)/2 \leq n \leq (N-1)/2$$

$$= 0 \quad \text{otherwise.}$$

Frequency response of a Blackmann window :-

$$W_B(e^{j\omega}) = 0.4 \frac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}} + 0.25 \frac{\sin(\omega N/2 - N\pi/N-1)}{\sin(\omega/2 - \pi/N-1)}$$

$$+ 0.25 \frac{\sin(\omega N/2 + N\pi/N-1)}{\sin(\omega/2 + \pi/N-1)}$$

$$+ 0.04 \frac{\sin(\omega N/2 + 2N\pi/N-1)}{\sin(\omega/2 + 2\pi/N-1)}$$

$$+ 0.04 \frac{\sin(\omega N/2 - 2N\pi/N-1)}{\sin(\omega/2 - 2\pi/N-1)}.$$

Equation :-

$$\omega_B(n) = 0.42 + 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1} \quad \text{for } -(N-1)/2 \leq n \leq (N-1)/2.$$

$$= 0 \quad \text{otherwise.}$$

Equation for kaiser window:-

$$w_k(n) = I_0\left[\frac{\alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2}}{I_0(\alpha)}\right] \text{ for } |n| \leq \frac{N-1}{2}$$

$$= 0 \quad \text{otherwise.}$$

where $\alpha \rightarrow$ independent parameter.

$I_0(x)$ is the zeroth order Bessel function of the first kid

$$I_0(x) = 1 + \sum_{k=1}^{\alpha}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

Advantages of kaiser window:-

1. It provides flexibility for the designer to select the side lobe level and N.
2) It has the attractive property that the side lobe level can be varied continuously from the low value in the Blackman window to the high value in the rectangular window.

Comparison between Hamming and kaiser window

	Hamming Window	Kaiser Window
1.	The main lobe width is equal to $\frac{8\pi}{N}$ and the peak side lobe level is $-41dB$	The main lobe width, the peak side lobe level can be varied by varying the parameters α and N.
2.	The low pass filter designed will have first side lobe peak of $-53dB$.	The side lobe peak can be varied by the varying the parameter α

1) Design a filter with

$$H_d(e^{j\omega}) = e^{-j3\omega} \quad -\pi/4 \leq \omega \leq \pi/4$$

$$= 0 \quad \pi/4 < |\omega| \leq \pi$$

Using a Hamming window with $N = 7$.

Soln:-

Given $H_d(e^{j\omega}) = e^{-j3\omega}$.

The frequency response is having a term $e^{-j\omega(N-1)/2}$ which gives $h(n)$ symmetrical about $n = \frac{N-1}{2} = 3$.

we have

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-j3\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j(n-3)\omega}\, d\omega.$$

$$= \frac{\sin \pi/4\,(n-3)}{\pi(n-3)}. \quad \text{(causal sequence)}$$

For $N=7 \Rightarrow$

$$h_d(0) = h_d(6) = 0.075$$

$$h_d(1) = h_d(5) = 0.159$$

$$h_d(2) = h_d(4) = 0.22$$

$$h_d(3) = 0.25$$

The non-causal window sequence is

$$W_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -(N-1)/2 \le n \le (N-1)/2$$

$$= 0 \quad \text{otherwise.}$$

For $N=7 \Rightarrow$

$$WH_n(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1} \quad \text{for } -3 \le n \le 3$$

$$= 0 \quad \text{otherwise.}$$

$n = 0 \Rightarrow$

$W_{Hn}(0) = 0.5 + 0.5 = 1.$

$W_{Hn}(-1) = W_{Hn}(1) = 0.5 + 0.5 \cos\frac{\pi}{3} = 0.75$

$W_{Hn}(-2) = W_{Hn}(2) = 0.5 + 0.5 \cos\frac{2\pi}{3} = 0.25$

$W_{Hn}(-3) = 0.5 + 0.5 \cos\pi = 0.$

The causal window sequence can be obtained by shifting the sequence $W_{Hn}(n)$ to right by 3 samples.

$W_{Hn}(0) = W_{Hn}(6)$; $W_{Hn}(1) = W_{Hn}(5) = 0.25$
$= 0$

$W_{Hn}(2) = W_{Hn}(4) = 0.75$ & $W_{Hn}(3) = 1.$

The filter coefficients using Hanning window are for $0 \leq n \leq 6$.

$h(n) = h_d(n) W_{Hn}(n)$ for $0 \leq n \leq 6$.

$h(0) = h(6) = h_d(0) W_{Hn}(0) = (0.075)(0) = 0.$

$h(1) = h(5) = h_d(1) W_{Hn}(1) = (0.159)(0.25) = 0.03975$

$h(2) = h(4) = h_d(2) W_{Hn}(2) = (0.22)(0.75) = 0.165$

$h(3) = h_d(3) W_{Hn}(3) = (0.25)(1) = 0.25$ //

H.W:

1. Design the following filters using Fourier series method. Take $N = 7$.

1. Low pass filter $H(e^{j\omega}) = 1$ for $0 \leq |\omega| \leq \pi/6$

$= 0$ otherwise.

2. High pass filter $H(e^{j\omega}) = 1$ for $\pi/6 \leq |\omega| \leq \pi$

$= 0$ otherwise.

3. Band stop filter $= 1$ for $0 \leq |\omega| \leq \pi/6$ and

$\pi/3 \leq \omega \leq \pi$

$= 0$ otherwise.

4. Design a filter with

$H_d(e^{j\omega}) = e^{-j5\omega}$ $\quad -\pi/2 \leq \omega \leq \pi/2$.

$= 0$ $\quad \pi/2 < |\omega| \leq \pi$

using Blackman window with $N = 11$.

1. Design an FIR Lowpass filter satisfying the following specifications.

$\alpha_s \geq 44.0\,dB$ $\quad \omega_{sf} = 100\,rad/sec$

$\alpha_p \leq 0.1\,dB$

$\omega_s = 30\,rad/sec$

$\omega_p = 20\,rad/sec$

Soln:- From the given specifications

$$B = \omega_s - \omega_p = 10\,rad/sec.$$

$$\omega_c = \frac{1}{2}(\omega_p + \omega_s) = 25\,rad/sec.$$

$$\omega_c(\text{in radians}) = \omega_c T = \omega_c \frac{2\pi}{\omega_{sf}}$$

$$= (25)\frac{(2\pi)}{100} = \pi/2.$$

Step 1:-

$$H(e^{j\omega}) = 1 \text{ for } |\omega| \leq \pi/2$$

$$= 0 \text{ for } \pi/2 < |\omega| \leq 2\pi$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n} d\omega = \frac{\sin \frac{\pi}{2} n}{\pi n}$$

Step 2:

$$\delta_1 = 10^{-0.05(44)} = 6.3096 \times 10^{-3}$$

$$\delta_2 = \frac{10^{+0.05(0.1)} - 1}{10^{+0.05(0.1)} + 1}$$

$$= 5.7563 \times 10^{-3}$$

$\delta = \min\{\delta_1, \delta_2\}$

$= 5.7563 \times 10^{-3}$

Step 3:- $\alpha_s = -20\log_{10}\delta = 44.797\,dB$.

Step 4:- for $\alpha_s = 44.797\,dB$

$= 0.5842(\alpha_s - 21)^{0.4} + 0.07886(\alpha_s - 21)$

$= 3.9524$.

Step 5: for $\alpha_s = 44.797\,dB$

$$D = \frac{\alpha_s - 7.95}{14.36} = 2.5666.$$

Step 6:

$$N \geq \frac{\omega_{sf} D}{B} + 1.$$

$$\geq \frac{100(2.566)}{10} + 1$$

≥ 26.66

Hence $\boxed{N = 27}$

Step 7:- Window Sequence

$$w_K(n) = \frac{I_0\left(\alpha\sqrt{1 - \left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha)}.$$

for $|n| \leq \frac{N-1}{2}$. For

$\alpha = 3.9524$

We can find

$$I_0(\alpha) = 10.8468$$

$$\omega_k(0) = \frac{I_0(\alpha)}{I_0(\alpha)} = 1$$

$$\omega_k(1) = \omega_k(-1) = \frac{I_0(3.9406)}{10.8468}$$

$$= \frac{10.7379}{10.8468} = 0.9899$$

$$\omega_k(2) = \omega_k(-2) = \frac{I_0(3.9073)}{10.8468} = \frac{10.4163}{10.8468} = 0.9603$$

$$\omega_k(3) = \omega_k(-3) = \frac{I_0(3.8457)}{10.8468} = \frac{9.8964}{10.8468} = 0.9124$$

$$\omega_k(4) = \omega_k(-4) = \frac{I_0(3.7606)}{10.8468} = \frac{9.2018}{10.8468} = 0.84835$$

$$\omega_k(5) = \omega_k(-5) = \frac{I_0(3.623)}{10.8468} = \frac{8.1856}{10.8468} = 0.75465$$

$$\omega_k(6) = \omega_k(-6) = \frac{I_0(3.5062)}{10.8468} = \frac{7.4168}{10.8468} = 0.68378$$

$$\omega_k(7) = \omega_k(-7) = \frac{I_0(3.3305)}{10.8468} = \frac{6.4025}{10.8468} = 0.59027$$

$$\omega_k(8) = \omega_k(-8) = \frac{I_0(3.1154)}{10.8468} = \frac{5.3613}{10.8468} = 0.49428$$

$$\omega_k(9) - \omega_k(-9) = \frac{I_0(2.852)}{10.8468} = \frac{4.3311}{10.8468} = 0.3995$$

$$w_k(10) = w_k(-10) = \frac{I_0(2.5257)}{10.8468} = \frac{3.3553}{10.8468} = 0.30934$$

$$w_k(11) = w_k(-11) = \frac{I_0(2.1063)}{10.8468} = \frac{2.4574}{10.8468} = 0.2265$$

$$w_k(12) = w_k(-12) = \frac{I_0(1.52)}{10.8468} = \frac{1.6666}{10.8468} = 0.1536$$

$$w_k(13) = w_k(-13) = \frac{I_0(0)}{10.8468} = \frac{1}{10.8468} = 0.0922$$

The impulse response $h_d(n)$ & $h(n)$ are given below

n	$h_d(n)$	$h(n) = h_d(n)\, w_k(n)$
0	0.5	0.5
1	0.318	0.31479
2	0	0.
3	−0.106	−0.0967
4	0	0
5	0.06366	0.04804
6	0	0
7	−0.0454	−0.0268
8	0	0
9	0.03536	0.014126
10	0	0
11	−0.0289	−0.006546.
12	0	0
13	0.02448	0.002267

The transfer function is given by

$$H(z) = z^{-13}\left[h(0) + \sum_{n=0}^{13} h(n)\,(z^{n} + z^{-n})\right]$$

H.W:

1) Design a FIR bandpass digital filter satisfying the following specifications.

$f_{p1} = 20\,Hz$ $\alpha_p = 0.5\,dB$

$f_{p2} = 30\,Hz$ $\alpha_s = 30\,dB$.

2) Design an FIR lowpass filter satisfying the following specifications.

$\alpha_p \leq 0.5\,dB$ $\alpha_s \geq 31\,dB$.

$\omega_p = 10\,rad/sec$ $\omega_s = 25\,rad/sec$

$\omega_{sf} = 100\,rad/sec$.

Frequency sampling method of designing FIR filters

1. Determine the filter coefficients $h(n)$ obtained by sampling

$$H_d(e^{j\omega}) = e^{-j(N-1)\omega/2} \quad 0 \leq |\omega| \leq \pi/2$$

$$= 0 \quad \pi/2 \leq |\omega| \leq \pi.$$

for $N = 7$

Soln: Given $N = 7$

$k = 0, 1, 2, \ldots, 6.$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{7}}$$

$$|H(k)| = 1 \quad \text{for } k = 0, 1, 6.$$

$$= 0 \quad \text{for } k = 2, 3, 4, 5.$$

We know

$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k = -\frac{6}{7}\pi k \text{ for } k = 0, 1, 2, 3.$$

$$= (N-1)\pi - \left(\frac{N-1}{N}\right)\pi k = 6\pi - \frac{6\pi k}{7}$$

$$= \frac{6\pi}{7}(7-k) \text{ for } k = 4, 5, 6.$$

Frequency response of linear phase filter =>

$$H(k) = e^{-j6\pi k/7} \quad k = 0, 1$$

$$= 0 \quad \text{for } k = 2, 3, 4, 5 .$$

$$= e^{-j6\pi(k-7)/7} \quad \text{for } k = 6.$$

The filter coefficients for N odd are given by

$$h(n) = \frac{1}{N} \left\{ H(0) + 2 \sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k) e^{j2\pi kn/7} \right] \right\}$$

$$n = 0, 1, \cdots, N-1$$

$$= \frac{1}{7} \left\{ 1 + 2 Re \left(e^{-6\pi/7} e^{j2\pi kn/7} \right) \right\}$$

$$= \frac{1}{7} \left\{ 1 + 2 Re \left(e^{j2\pi(n-3)/7} \right) \right\}$$

$$= \frac{1}{7} \left\{ 1 + 2 \cos \frac{2\pi}{7} (n-3) \right\} .$$

$$h(0) = h(6) = \frac{1}{7} \left(1 + 2 \cos \frac{6\pi}{7} \right) = -0.11456$$

$$h(1) = h(5) = \frac{1}{7} \left(1 + 2 \cos \frac{4\pi}{7} \right) = 0.07928$$

$$h(2) = h(4) = \frac{1}{7} \left(1 + 2 \cos \frac{2\pi}{7} \right) = 0.321$$

$$h(3) = \frac{1}{7} (1+2) = 0.42857 .$$

2. Determine the coefficients of a linear phase FIR filter of length $M = 15$ has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H\left(\frac{2\pi k}{15}\right) = 1 \quad k = 0, 1, 2, 3.$$

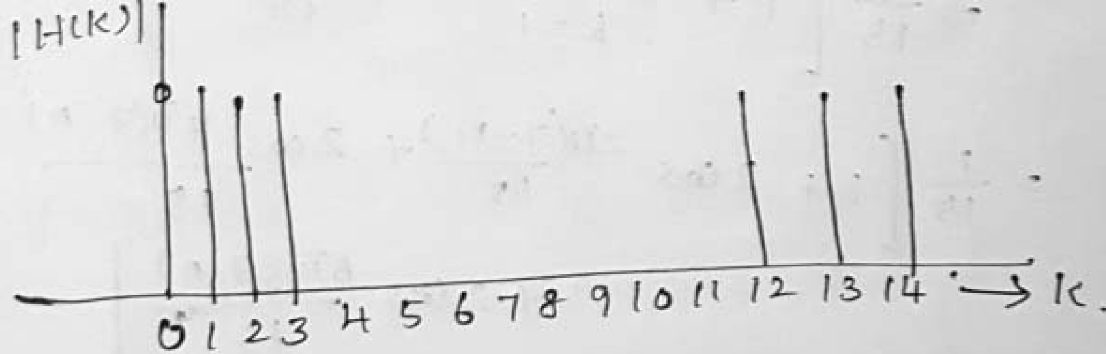
$$= 0 \quad k = 4, 5, 6, 7.$$

Soln:-

$$|H(k)| = 1 \text{ for } 0 \leq k \leq 3 \text{ and } 12 \leq k \leq 14$$

$$= 0 \text{ for } 4 \leq k \leq 11.$$

Ideal magnitude response =).



$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k.$$

$$= \frac{-14}{15}\pi k \quad 0 \leq k \leq 7.$$

and

$$\theta(k) = 14\pi - \frac{14\pi k}{15} \quad \text{for } 8 \leq k \leq 14.$$

$$H(k) = e^{-j14\pi k/15} \quad \text{for } k = 0, 1, 2, 3.$$

$$= 0 \quad \text{for } 4 \leq k \leq 11.$$

$$= e^{-j14\pi(k-15)/15} \quad \text{for } 12 \leq k \leq 14.$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)\, e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} Re\left(e^{-j14\pi k/15} e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$; $h(1) = h(3) = 0.041$

$h(4) = h(10) = -0.1078$; $h(2) = h(12) = 0.0666$

$h(3) = h(11) = -0.0365$; $h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$; $h(7) = 0.466$.

3. Using frequency sampling method, design a bandpass filter with the following specifications.

Sampling frequency F = 8000 Hz.

cutoff frequencies f_{c_1} = 1000 Hz.

f_{c_2} = 3000 Hz.

Determine the filter coefficient N = 7.

Soln:-

$$\omega_{c_1} = 2\pi f_{c_1} T = \frac{2\pi f_{c_1}}{F} = \frac{2\pi(1000)}{8000} = \frac{\pi}{4}.$$

$$\omega_{c_2} = 2\pi f_{c_2} T = \frac{2\pi f_{c_2}}{F} = \frac{2\pi(3000)}{8000} = \frac{3\pi}{4}.$$

$$H(k) = H_d(e^{j\omega})\Big|_{\omega = \frac{2\pi}{7}k} \quad k = 0, 1, \ldots, 6.$$

$$|H(k)| = 0 \quad \text{for } k = 0, 3$$

$$= 1 \quad \text{for } k = 1, 2$$

$$\theta(k) = -\left[\frac{N-1}{N}\right]\pi \quad \text{for } 0 \le k \le \frac{N-1}{2}$$

$$= -\frac{6}{7}\pi k. \quad \text{for } 0 \le k \le 3$$

$$H(k) = 0 \quad \text{for } k = 0, 3$$

$$= e^{-j6\pi k/7}. \quad \text{for } k = 1, 2.$$

The filter coefficients are given by

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left[H(k) e^{j2\pi kn/N}\right]\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{3} Re\left(e^{-j6\pi k/7} e^{j2\pi kn/7}\right)\right]$$

$$= \frac{1}{7}\left[2\sum_{k=1}^{2} \cos\frac{2\pi k}{7}(3-n)\right]$$

$$= \frac{2}{7}\left[\cos\frac{2\pi}{7}(3-n) + \cos\frac{4\pi}{7}(3-n)\right]$$

$h(0) = h(6) = -0.07928$

$h(1) = h(5) = -0.321$

$h(2) = h(4) = 0.11456$

$h(3) = 0.57.$

Comparison between FIR and IIR filters

	FIR filter	IIR filter
1.	The impulse response of this filter is restricted to a finite number of samples.	The impulse response of this filter extends over an infinite duration.
2.	FIR filters can have precisely linear phase.	These filters do not have linear filters.
3.	Closed form design equations do not exist.	A variety of frequency selective filters can be designed using closed-form design formulas.
4.	Most of the design methods are iterative procedures, requiring powerful computational facilities for their implementation	These filters can be designed using only a hand calculator and tables of analog filter design parameters.
5.	Greater flexibilities to control the shape of their magnitude response.	Less flexibility specially for obtaining non-standard frequency responses.

6.	In these filters, the poles are fixed at the origin, high selectivity can be achieved by using a relatively high order for the transfer function.	The poles are placed anywhere inside the unit circle, high selectivity can be achieved with low-order transfer functions.
7.	Always stable	Not always stable.
8.	Errors due to roundoff noise are less severe.	IIR filters are more susceptible to errors due to roundoff noise.

HW:

1) Using frequency sampling method design a band reject filter with the following specifications.

Sampling frequency $F = 10 kHz$

cutoff frequency $f_{c1} = 2000 kHz$

$f_{c2} = 4000 kHz$

Determine the filter coefficients for $N = 7$.

2) Design an FIR filter approximating the ideal frequency response

$$H_d(e^{j\omega}) = e^{-j\alpha\omega} \quad \text{for } |\omega| \leq \pi/6$$

$$= 0 \quad \text{for } \pi/6 \leq |\omega| \leq \pi$$

Determine the filter coefficients for $N = 13$.

3) Using a rectangular window technique design a low pass filter with passband gain of unity, cutoff frequency of 1000 Hz and working at a sampling frequency of 5 kHz. The length of impulse response should be '7'.

IV unit.

Fixed point and floating point number representations - comparison - Truncation and Rounding errors - Quantization noise - derivation for quantization noise power - Coefficient quantization error - Product quantization error - Overflow error - Round off noise power - limit cycle oscillations due to product round off and overflow errors - signal scaling.

Fixed point and floating point number representations :-

Different types of arithmetic in digital systems.

Three types: Three common formats that are used to represent the numbers in a digital computer arithmetic

(i) Fixed point representation.

(ii) Floating point representation.

(iii) Block Floating point representation.

(i) Fixed point representation:-

In fixed point arithmetic the position of the binary point is fixed. The bit to the right represent the fractional part of the number and those ~~left~~ to the left represent the image part. Ex: binary number 01.1100
⇓
decimal value $(1.75)_{10}$.

Three forms: (Depending on the way -ve numbers are represented).

1. Sign-magnitude
2. 1's complement
3. 2's complement.

1. Sign-magnitude:- For sign-magnitude representation the leading binary digit is used to represent the sign. If it is equal to 1, the number is negative, otherwise the number is positive

Ex: $(-1.75)_{10} \Rightarrow 11.110000 \Rightarrow$ Ans.

$1.75 \Rightarrow 01.110000$.

In sign magnitude form the number '0' has two representations i.e. 00.000000 (or) 10.000000

with b bits only $2^b - 1$ numbers can be represented

2) **1's complement:-** In one's complement form, the positive number is represented as in the sign magnitude system. To obtain the negative of a positive number, one simply complements all the bits of the positive number.

Ex: the negative of $01.110000 \Rightarrow (10.001111)_2$

The number '0' has two representations i.e. 00.000000 and 11.111111 in a 1's complement representation.

3) **2's complement:-** In two's complement representation positive numbers are represented as in sign magnitude and one's complement. The negative number is obtained by complementing all the bits of the positive number and

adding one to the LSB.

Ex1: $(0.5625)_{10} = (0.100100)_2$

$$-(0.5625)_{10} = 1.011011$$

$$\begin{array}{r} 1.011011 \\ 0.000001 \\ \hline (1.011100)_2 \end{array}$$

Ex2:

$$(0.875)_{10} = (0.11100)_2$$

$$(-0.875)_{10} = (1.00100)_2$$

ie.

$$(0.875)_{10} = (0.111000)_2$$

$$\begin{array}{rl} 1.000111 & \text{complementing each bit.} \\ 0.000001 & \text{Add 1 to Least Significant Bit (LSB)} \\ \hline (-0.875)_{10} = 1.000100 & \end{array}$$

ie. The magnitude of the negative number is given by

$$\boxed{1 - \sum_{i=1}^{b} c_i \cdot 2^{-i}}$$

ie.

$$\boxed{1 - 2^{-3} = 0.875}$$

Addition of two fixed point numbers:-

The two numbers are added bit by bit starting from right, with carry bit being added to the next bit.

Ex 1:- $(0.5)_{10} = 0.100_2$

$(0.125)_{10} = 0.001_2$

$$\underline{0.101_2} = (0.625)_{10}$$

↓ sign bit.

Ex 2: $(0.5)_{10} = 0.100_2$

$(0.625)_{10} = 0.101_2$

$$\underline{1.001_2} = (-0.125)_{10}$$

↳ Sign bit.

Subtraction of two fixed point numbers -

Ex 1.-

Decimal	Two's complement	
0.5	0.100	} add
−0.25	1.110	

$10.010 = 0.25$

↳ neglect carry bit

$0.010 \Rightarrow (0.25)_{10}$

Ex 2: Decimal Two's complement.

Decimal		Two's complement
0.25	=	0.010
−0.5	=	1.100

} add.

1.110 = (−0.25)$_{10}$.

'1' so it is negative.

To get decimal, two's complement

$$\begin{array}{r} 0.001 \\ 1 \\ \hline 0.010 \end{array} = (-0.25)_{10}.$$

Multiplication in fixed point arithmetic:-

In multiplication of two fixed point numbers first the sign and magnitude components are separated.

The magnitude of the numbers are multiplied first, then the sign of the product is determined and applied to the result. ie. b (bit) $\times b$ (bit) $= 2b$ bits

ie. $b = b_i + b_j$.

Ex: $(11)_2 \times (11)_2 = (1001)_2$

0.1001 × 0.0011 = 0.00011011

0.1001 → 4 bits, 0.0011 → 4 bits, 0.00011011 → 8 bits.

(ii) Floating point arithmetic: In floating point representation a positive number is represented as $F = 2^C . M$ where M, called Mantissa, is a function such that $1/2 \leq M < 1$ and C, the exponent can be either positive or negative.

The decimal number 2.25, 0.75 have floating point represented as

$$2.25 = 2^2 \times 0.5625 = 2^{010} \times 0.1001$$

and $0.75 = 2^0 \times 0.75 = 2^{000} \times 0.1100$ respectively.

Negative floating point numbers are generally represented by considering the mantissa as a fixed point number. The sign of the floating point number is obtained from the first bit of mantissa.

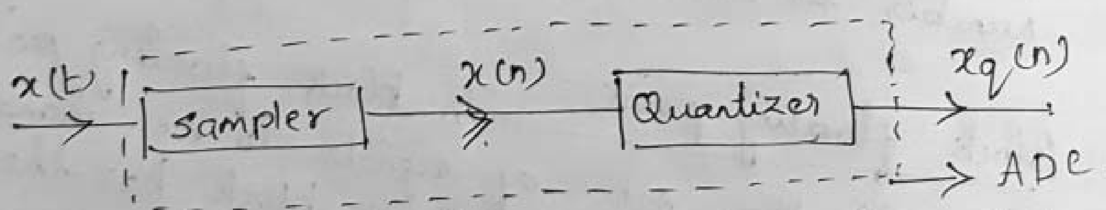
(iii) Block floating point:- In block floating point arithmetic the set of signals to be handled is divided into blocks. Each block has the same value for the exponent. The arithmetic operations within the block use fixed point arithmetic and only one exponent per block is stored thus saving memory. This representation of numbers is most suitable in certain FFT flow graphs and in digital audio applications;

Advantages of floating point arithmetic:

1. Larger dynamic range
2. Overflow in floating point representation is unlikely.

Quantization noise:-

The continuous time signal is to be converted into digital by using ADC. A digital signal processor contains a device, A/D converter that operates on the analog input $x(t)$ to produce $x_q(n)$ which is binary sequence of 0's and 1's.



At first the signal is sampled at regular intervals to produce a sequence $x(n)$ of infinite precision. Each sample $x(n)$ is expressed in terms of finite number of bits giving the sequence $x_q(n)$. The difference signal $e(n) = x_q(n) - x(n)$ is called A/D conversion noise or quantization noise.

Two quantization methods:-

1. Truncation
2. Rounding.

Quantization step size:- Let us assume a sinusoidal signal varying between +1 and −1 having a dynamic range 2. If ADC used to convert the sinusoidal signal employs b+1 bits including sign bit, the number levels available for quantizing $x(n)$ is 2^{b+1}. Thus the interval between successive levels.

$$q = \frac{2}{2^{b+1}} = 2^{-b}$$

where 'q' is known as quantization step size.

Truncation:- Truncation is the process of discarding all bits less significant than least significant bit that is retained.

Suppose we truncate the following numbers from 7 bits to 4 bits, we get 0.0011001 to 0.0011 and 0.0100100 to 0.0100.

For truncation in floating point systems the effect is seen only in mantissa. If the mantissa is truncated to b bits, then the error satisfies

$$0 \geq \epsilon > -2 \cdot 2^{-b} \text{ for } x > 0 \text{ and}$$

$$0 \leq \epsilon < -2 \cdot 2^{-b} \text{ for } x < 0$$

for 2's complement representation of the mantissa.
If the mantissa is represented by 1's complement or sign magnitude, then the error satisfies

$$-2 \cdot 2^{-b} < \epsilon \leq 0 \quad \text{for all } x$$

Here $\epsilon = \dfrac{x_T - x}{x}$

where x_T is the truncated value of x.

Relationship between truncation error 'e' and the bits 'b' for representing a decimal into binary

For a 2's complement representation, the error due to truncation for both positive and negative values of x is $0 \geq x_T - x > -2^{-b}$ where 'b' is the number of bits and x_T is the truncated value of x.
The equations hold for both sign-magnitude, 1's complement if $x > 0$.
If $x < 0$, then for sign-magnitude and for 1's complement the truncation error satisfies

$$0 \leq x_T - x < 2^{-b}.$$

Rounding:- Rounding a number to b bits is accomplished by choosing the rounded result as the b bit number closest to the original number unrounded. For fixed point arithmetic, the error made by rounding a number to bits satisfies the inequality

$$-\frac{2^{-b}}{2} \leq x_T - x \leq \frac{2^{-b}}{2}$$

for all three types of number systems, i.e. two's complement, one's complement and sign-magnitude.

For floating point numbers the error made by rounding a number to b bits satisfies the inequality.

$$-2^{-b} \leq \epsilon \leq 2^{-b} \quad \text{where } \epsilon = \frac{x_T - x}{x}.$$

Quantization errors:

In digital computer coefficients and numbers are stored in finite-length registers. Therefore, coefficients and numbers are quantized by truncation or rounding off when they are stored.

The following errors arise due to quantization of numbers.

1. Input quantization error.
2. Product quantization error.
3. Coefficient quantization error.

1. Input quantization error:-

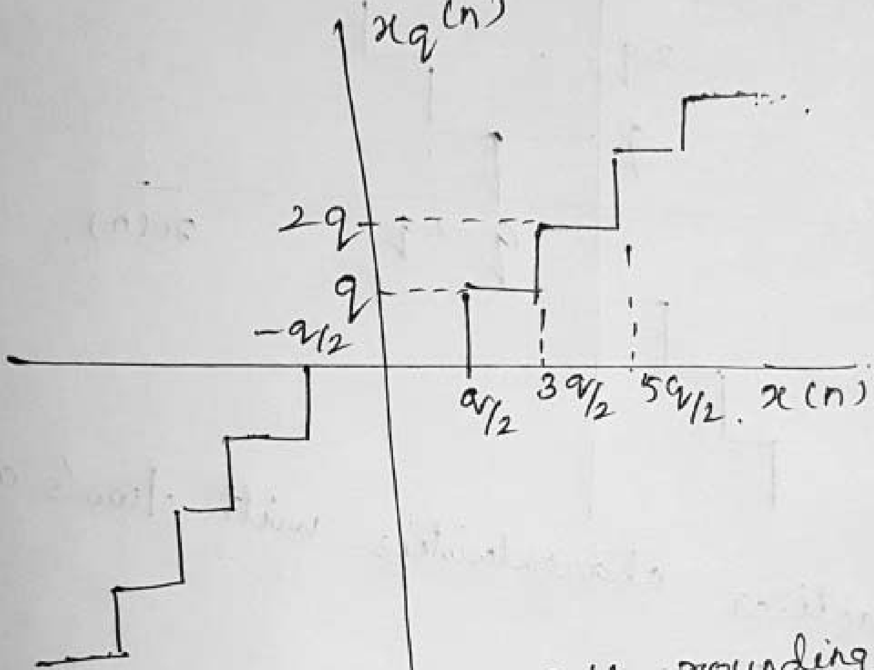
In digital signal processing, the continuous time input signals are converted into digital using a b-bit ADC. The representation of continuous signal amplitude by a fixed digit produces an error, which is known as quantization error.

$$e(n) = x_q(n) - x(n).$$

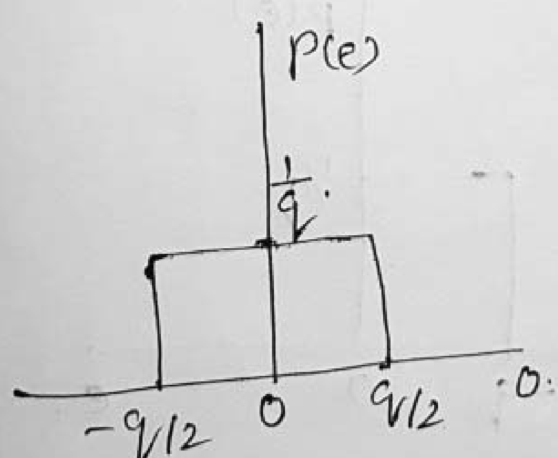
where $x_q(n)$ = Sampled quantized value and
$x(n)$ = Sampled unquantized value.

The other type of quantization can be obtained by truncation. In truncation the signal is represented by the highest quantization level that is not greater than the signal. ∴ The two's complement truncation, the error $e(n)$ is always negative and satisfies the inequality,

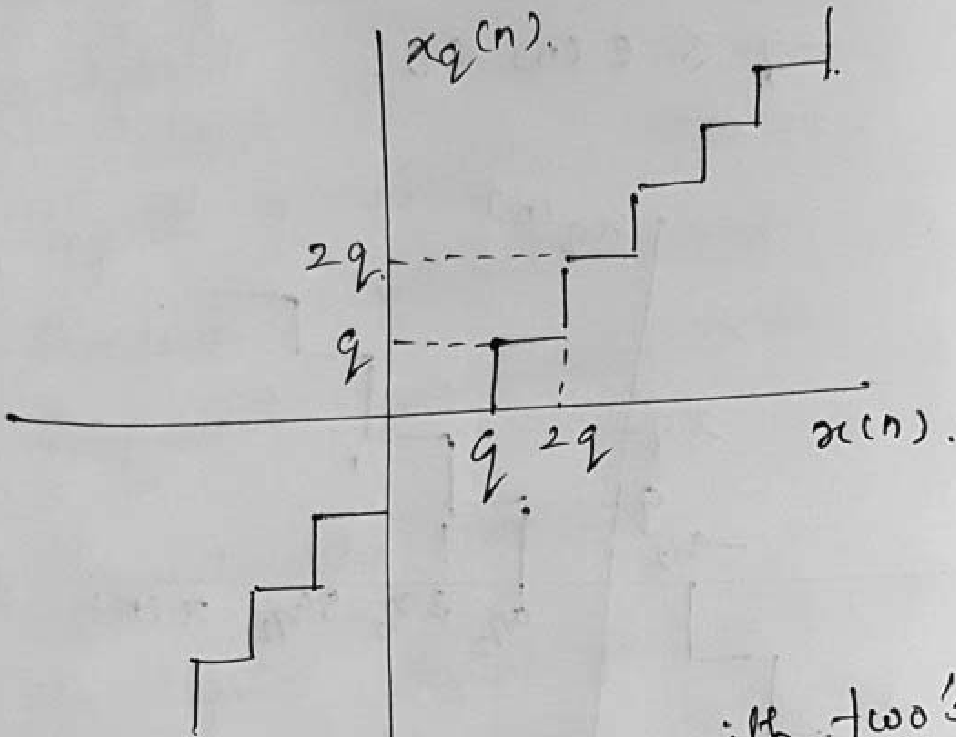
$$-q \leq e(n) < 0.$$



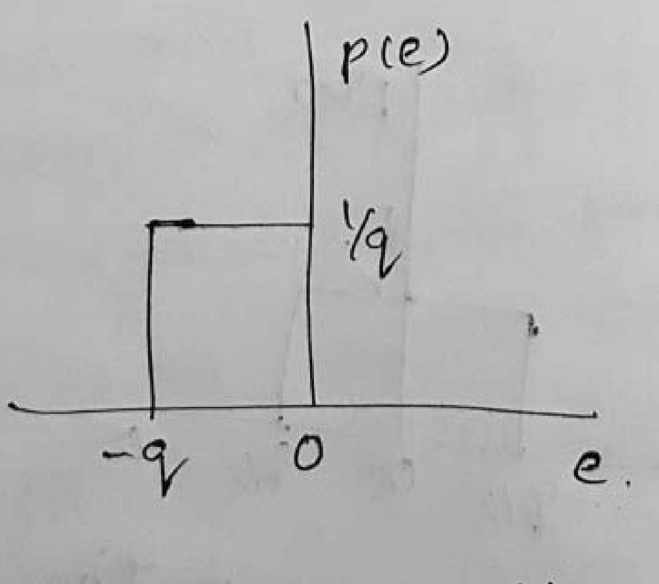
a) Quantizer characteristics with rounding.



b) Probability density function.



c) Quantizer characteristics with two's complement truncation.

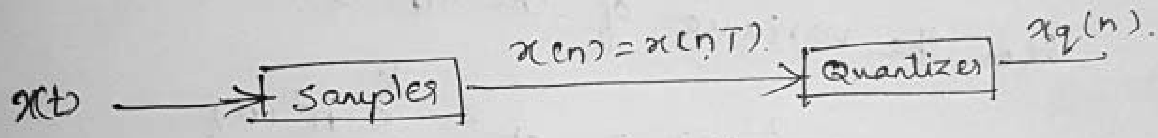


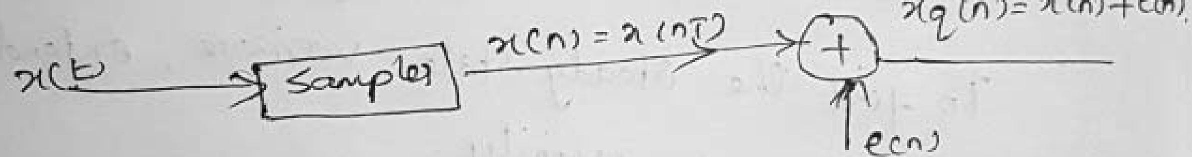
d) Probability density function of truncation error.

Steady state Input noise power:-

In digital processing of analog signals, the quantization error is commonly viewed as an additive noise signal. that is

$$x_q(n) = x(n) + e(n).$$

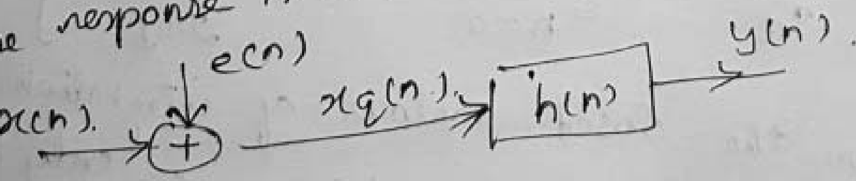




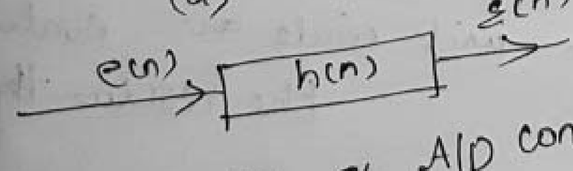
∴ The A/D converter o/p is the sum of the input signal $x(n)$ and the error signal $e(n)$.

Steady State output noise power:-

Due to A/D conversion noise, represent the quantized input to a digital system with impulse response $h(n)$.



(a)



Representation of A/D conversion noise.

Let e(n) be the output noise due to quantization of the input. Then

$$\varepsilon(n) = e(n) * h(n)$$

$$= \sum_{k=0}^{n} h(k)\, e(n-k).$$

Then the variance of the o/p =).

$$\sigma_\epsilon^2(n) = \sigma_\epsilon^2 \sum_{n=0}^{k} h^2(n).$$

To find the steady state variance, extend the limit k upto infinity.

Then

$$\sigma_\epsilon^2 = \sigma_\epsilon^2 \sum_{n=0}^{\infty} h^2(n).$$

Using Parseval's theorm. the steady state o/p noise variance due to the quantization error is given by

$$\sigma_\epsilon^2 = \sigma_\epsilon^2 \sum_{n=0}^{\infty} h^2(n) = \frac{\sigma_\epsilon^2}{2\pi j} \oint H(z) H(z^{-1}) z^{-1} dz$$

where the closed contour of integration is around the unit circle $|z| = 1$ in which case, only the poles that lie inside the unit circle are evaluated using the residue threorem.

1. The output signal of an A/D converter is passed through a first order lowpass filter with transfer function is given by

$$H(z) = \frac{(1-a)z}{z-a} \text{ for } 0 < a < 1$$

Find the steady state output noise power due to quantization at the output of the digital filter.

<u>Soln:-</u> We have

$$\sigma_e^2 = \sigma_e^2 \cdot \frac{1}{2\pi j} \oint H(z) H(z^{-1}) z^{-1} dz \text{ --- (1)}$$

Given $$H(z) = \frac{(1-a)z}{z-a} \text{ --- (2)}$$

then $$H(z^{-1}) = \frac{(1-a)z^{-1}}{(z^{-1}-a)} \text{ --- (3)}$$

Substitute (2) & (3) in (1).

$$\sigma_e^2 = \sigma_e^2 \frac{1}{2\pi j} \oint \frac{(1-a)^2 z \cdot z^{-1} z^{-1} dz}{(z-a)(z^{-1}-a)}$$

$$= \sigma_e^2 \left[\text{residue of } H(z) H(z^{-1}) z^{-1} \text{ at } z = a. + \text{residue of } H(z) H(z^{-1}) z^{-1} \text{ at } z = \frac{1}{a} \right].$$

$$= \sigma_e^2 \left[(z-a) \frac{(1-a)^2 z^{-1}}{(z-a)(z^{-1}-a)} + 0 \right]$$

↳ residue at $z = \frac{1}{a}$ is equal to zero as $\frac{1}{a} > 1$.

$$= \sigma_e^2 \left[\frac{(1-a)^2}{1-a^2} \right] = \sigma_e^2 \left[\frac{(1-a)^2}{(1+a)(1-a)} \right]$$

$$= \sigma_e^2 \left[\frac{1-a}{1+a} \right]$$

where $\sigma_e^2 = \frac{2^{-2b}}{12}$.

H.W:

1) Find the steady state variance of the noise in the output due to quantization of input for the first order filter.

$$y(n) = a y(n-1) + x(n)$$

2. Product quantization error:-

It arises at the output of a multiplier. Multiplication of a b bit data with a 'b' bit coefficient results a product having 2b bits. Since a b-bit register is used, the multiplier output must be rounded or truncated to b bits, which produces an error. This error is known as product quantization error.

1. Draw the quantization noise model for a second order system $H(z) = \frac{1}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$. Find the steady state output noise variance.

Soln:-

Given

$$H(z) = \frac{1}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$$

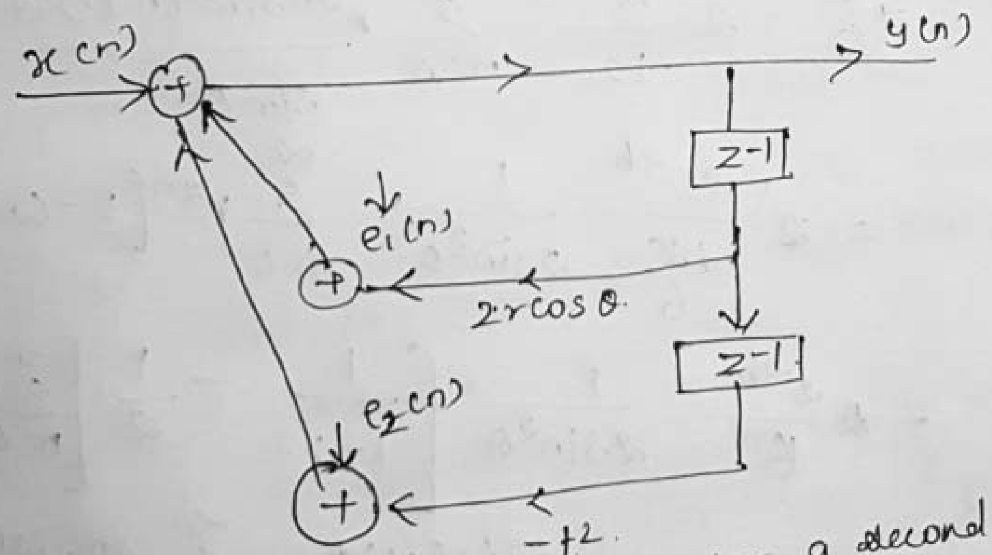


Fig: Quantization noise model for a second order system.

Both the noise sources see the same transfer function.

$$H(z) = \frac{1}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$$

The impulse response of the transfer function is given by

$$h(n) = r^n \frac{\sin(n+1)\theta}{\sin\theta} u(n).$$

Steady state o/p noise variance is

$$\sigma_0^2 = \sigma_{01}^2 + \sigma_{02}^2$$

but $\sigma_{01}^2 = \sigma_{02}^2 = \sigma_e^2 \sum_{n=-\infty}^{\infty} h^2(n)$ which gives

$$\sigma_0^2 = 2 \cdot \frac{2^{-2b}}{12} \sum_{n=0}^{\infty} \frac{r^{2n}\sin^2(n+1)\theta}{\sin^2\theta}$$

$$= \frac{2^{-2b}}{6} \frac{1}{2\sin^2\theta} \sum_{n=0}^{\infty} r^{2n}\left[1-\cos 2(n+1)\theta\right]$$

$$= \frac{2^{-2b}}{6} \frac{1}{2\sin^2\theta}\left[\frac{1}{1-r^2} - \frac{1}{2}\left[\sum_{n=0}^{\infty} r^{2n} e^{j2(n+1)\theta} + \sum_{n=0}^{\infty} r^{2n} e^{-j2(n+1)\theta}\right]\right]$$

$$= \frac{2^{-2b}}{6} \frac{1}{2\sin^2\theta}\left[\frac{1}{1-r^2} - \frac{1}{2}\left(\frac{e^{j2\theta}}{1-r^2e^{2j\theta}} + \frac{e^{-j2\theta}}{1-r^2e^{-2j\theta}}\right)\right]$$

$$= \frac{2^{-2b}}{6} \frac{1}{2\sin^2\theta} \left[\frac{(1+r^2)(1-\cos 2\theta)}{(1-r^2)(1-2r^2\cos 2\theta + r^4)} \right]$$

$$= \frac{2^{-2b}}{6} \frac{(1+r^2)}{(1-r^2)(1-2r^2\cos 2\theta + r^4)}.$$

3. Coefficient Quantization error:-

The filter coefficients are computed to infinite precision in theory. But in digital computation the filter coefficients are represented in binary and are stored in registers.

If a b bit register is used, the filter coefficients must be rounded or truncated to b bits, which produce an error.

Due to quantization of coefficients, the frequency response of the filter may differ appreciably from the desired response and sometimes the filter may actually ~~fa~~ fail to meet the desired specifications. If the poles of the unit circle, desired filter are closed to the unit circle, then those of the filter with quantized coefficients may lie just outside the unit circle, leading to unstability.

1. Consider a second order IIR filter with

$$H(Z) = \frac{1.0}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

Find the effect on quantization on pole locations of the given system function in direct form and in cascade form. Take $b = 3$ bits.

Soln:-

Direct form I:

$$H(z) = \frac{1}{1-0.95z^{-1}+0.225z^{-2}}$$

$$(0.95)_{10} = (0.1111001\cdots)_2$$

$$(-0.95)_{10} = (1.1111001\cdots)_2$$

After truncation we have $(1.111)_2 = -0.875$

IIIrly $(0.225)_{10} = (0.001110\cdots)_2$

After truncation we have $(0.001)_2 = 0.125$

$$\text{So } H(z) = \frac{1}{(1-0.875z^{-1}+0.125z^{-2})}$$

<u>Cascade form</u>.

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

$$(-0.5)_{10} = (1.100)_2$$

$$(0.45)_{10} = (1.01110\cdots)_2$$

After truncation we have.

$$(1.011)_2 = (-0.375)_{10}.$$

$$\text{So } H(z) = \frac{1}{(1-0.5z^{-1})(1-0.375z^{-1})}.$$

<u>Zero - Input Limit cycle oscillations</u>:

<u>Two kinds</u> of limit cycle behaviours in DSP.

(i) Zero input limit cycle oscillations

(ii) Overflow limit cycle oscillations.

Zero-input limit cycle oscillations :-

For an IIR filter, implemented with infinite precision arithmetic, the output should approach zero in the steady state if the input is zero and it should approach a constant value if the input is a constant. However, with an implementation using finite length registers an o/p can occur even with zero input if there is a non-zero initial condition on one of the registers. The output may be a fixed value or it may oscillate between finite positive and negative values. This effect is referred to as (zero-input) limit cycle oscillations and is due to the non-linear nature of the arithmetic quantization.

Overflow-oscillations :- The addition of two fixed point arithmetic numbers cause overflow when the sum exceeds the word size available to store the sum. This overflow caused by adder make the filter output to oscillate between maximum amplitude limits. Such limit cycles have been referred to as overflow oscillations.

Methods used to prevent overflow:-

(i) Saturation arithmetic
(ii) Scaling.

(i) Saturation arithmetic:- When the sum of two fixed point numbers exceeds the dynamic range, overflow occurs, which causes the output of the adder to oscillate between maximum amplitude limits. Such limit cycle has been referred to as overflow oscillations. One way to avoid overflow is to modify the adder characteristics so that it performs saturation arithmetic. Thus when the overflow is sensed, the sum of the adder is equal to the maximum value. But Saturation arithmetic causes undesirable signal correlation distortion due to non-linearity in the adder.

(ii) Signal Scaling:- Saturation arithmetic eliminates limit cycles due to overflow, but it causes undesirable signal distortion due to non-linearity of the clipper. In order to limit the amount of non-linear distortion, it is important to scale the input signal and the unit sample response between the input signal and the unit sample response between the input and any internal summing node in the system such that overflow becomes a rare event.

1. For a second order digital filter

$$H(z) = \frac{1}{1-2r\cos\theta\, z^{-1} + r^2 z^{-1}} \quad |r| < 1.0.$$

Draw the direct form II realization and find the scale factor S_0 to avoid overflow.

Soln: The realization of the second order filter is

$$S_0^2 = \frac{1}{I}, \text{ where } I = \frac{1}{2\pi j} \oint_c \frac{z^{-1} dz}{D(z) D(z^{-1})}$$

For the given problem

$$D(z) = \frac{1}{H(z)}.$$

$$\therefore I = \frac{1}{2\pi j} \oint_c \frac{z^{-1} dz}{D(z) D(z^{-1})}$$

$$= \frac{1}{2\pi j} \oint H(z) H(z^{-1}) z^{-1} dz.$$

$$= \sum_{m=-\infty}^{\infty} h^2(m) \quad (\because \text{Using Parseval's theorem}).$$

For the given

$$H(z) = \frac{1}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

$$h(n) = \frac{r^n \sin(n+1)\theta}{\sin\theta} u(n).$$

$$I = \sum_{n=-\infty}^{\infty} h^2(n).$$

$$= \sum_{n=0}^{\infty} \frac{r^{2n} \sin^2(n+1)\theta}{\sin^2\theta}$$

$$= \frac{1+r^2}{(1-r^2)(1-2r^2\cos 2\theta + r^4)}.$$

$$\text{Scale factor } S_0 = \frac{1}{\sqrt{I}} \,//.$$

2. Convert the following numbers into decimal:

(i) $(1110.01)_2 = (2^3 \times 1 + 2^1 \times 1 + 2^1 \times 1 + 2^0 \times 0) \cdot (0 \times 2^{-1} + 1 \times 2^{-2})$

$= (8+4+2+0).(0+0.25)$

$= (14.25)_{10}$

(ii) $(11011.1110)_2 = (2^4 \times 1 + 2^3 \times 1 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1) \cdot (2^{-1} \times 1 + 2^{-2} \times 1 + 2^{-3} \times 1 + 2^{-4} \times 0)$

$= (16+8+2+1).(0.5+0.25+0.125)$

$= (27.875)_{10}$

3. Convert the following decimal numbers into binary..

(i) $(20.675)_{10}$

	Remainders
$(20)_{10} = 20 \div 2 = 10$	0
$10 \div 2 = 5$	0
$5 \div 2 = 1$	1
$2 \div 2 = 1$	0.
$1 \div 2 = 0$	1

		Integer part
0.675×2	1.35	1
0.35×2	0.70	0
0.7×2	01.4	1
0.4×2	0.8	0
0.8×2	1.6	1
0.6×2	1.2	1
0.2×4	0.8	0

$$(20.675)_{10} = (10100.1010110\ldots)_2$$

(ii) $(120.75)_{10}$

Remainder

$(120)_{10}$ = $120 \div 2 = 60$ — 0

$60 \div 2 = 30$ — 0

$30 \div 2 = 15$ — 0

$15 \div 2 = 7$ — 1

$7 \div 2 = 3$ — 1

$3 \div 2 = 1$ — 1

$1 \div 2 = 0$ — 1

	Integer part
$0.75 \times 2 = 1.5$	1
$0.5 \times 2 = 1.0$	1

$\therefore$

$(120.75)_{10} = (1111000.11)_2$

4. The input to the System

$y(n) = 0.999y(n-1) + x(n)$ is applied to an ADC. What is the power produced by the quantization noise at the output of the filter if the input is quantized to (a) 8 bits (b) 16 bits.

Soln:

Given $y(n) = 0.999y(n-1) + x(n)$

Taking z-transform on both sides we have

$$Y(z) = 0.999z^{-1}Y(z) + X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.999z^{-1}}.$$

Taking Inverse z-transform

$$h(n) = (0.999)^n u(n)$$

The quantization noise power at the output of the digital filter is

$$\sigma_\epsilon^2 = \sigma_e^2 \sum_{k=0}^{\infty} h^2(k)$$

$$= \sigma_e^2 \sum_{k=0}^{\infty} (0.999)^{2k}.$$

$$= \sigma_\epsilon^2 \frac{1}{(1-0.999)^2} = \sigma_\epsilon^2 (500.25)$$

$$\boxed{\sigma_\epsilon^2 = \frac{2^{-2b}}{12} (500.25)}$$

(a) Given : 8 bits

$b+1 = 8$ bits (Assuming including sign bit)

Then $\sigma_\epsilon^2 = \frac{2^{-14}}{12} (500.25) = 2.544 \times 10^{-3}$

(b) Given : 16 bits

$b+1 = 16$ bits

Then $\sigma_\epsilon^2 = \frac{2^{-30}}{12} (500.25)$

$= 3.8882 \times 10^{-8}$

5. Find the effect of coefficient quantization on pole locations of the given second order IIR system. When it is realized in direct form I and in cascade form. Assume a word length of 4 bits through truncation.

$$H(z) = \frac{1}{1 - 0.9z^{-1} + 0.2z^{-2}}$$

Soln:-

Direct form I :-

Let b = 4 bits including a sign bit.

$(0.9)_{10} = (0.111001\ldots)_2$

	Integer part
0.9 × 2 = 1.8	1
0.8 × 2 = 1.6	1
0.6 × 2 = 1.2	1
0.2 × 2 = 0.4	0
0.4 × 2 = 0.8	0
0.8 × 2 = 1.6	1

After truncation

We get.

$(0.111)_2 = (0.875)_{10}$.

$(0.2)_{10} = (0.00110\ldots)_2$

Integer part

$0.2 \times 2 = 0.4 \rightarrow 0$

$0.4 \times 2 = 0.8 \rightarrow 0$

$0.8 \times 2 = 1.6 \rightarrow 1$

$0.6 \times 2 = 1.2 \rightarrow 1$

$0.2 \times 2 = 0.4 \rightarrow 0$

After truncation we get

$(0.001)_2 = (0.125)_{10}$

The system function after coefficient quantization is

$$H(z) = \frac{1}{(1 - 0.875z^{-1} + 0.125z^{-2})}$$

Now the pole locations are given by

$z_1 = 0.695$

$z_2 = 0.1798$

Cascade form

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.4z^{-1})}$$

$(0.5)_{10} = (0.1000)_2$

After truncation we get

$$(0.100)_2 = (0.5)_{10}$$

After truncation we get $(0.011)_2 = (0.375)_{10}$.

$(0.4)_{10}$

	Integer part
$0.4 \times 2 = 0.8$	0
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	0.

The system function after coefficient quantization is

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.375z^{-1})}$$

The pole locations are given by

$$z_1 = 0.5$$

$$z_2 = 0.375$$

$$(0.4)_{10} = (0.01100\ldots)_2$$

UNIT-V Multirate Digital Signal Processing

- systems that employ multiple sampling rates in the processing of digital signals are called multirate digital signal processing systems.
- Multirate systems are sometimes used for sampling-rate conversion

In most applications multirate systems are used to improve the performance, or for increased computational efficiency.

- The basic Sampling operations in a multirate system are:

Decimation

Decreasing the sampling rate of signal

Interpolation

Increasing the sampling rate of signal

Sampling Rate Reduction by Integer Factor D

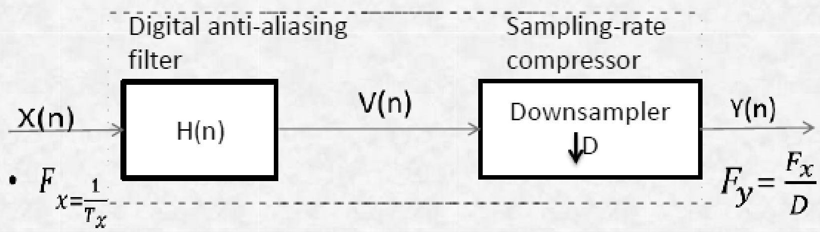
- Decimation by a factor of D, where D is a positive integer, can be performed as a two-step process, consisting of an anti-aliasing filtering followed by an operation known as downsampling

$$Y(n) = v(nD)$$
$$= \sum_{k=0}^{\infty} h(k)x(nD-k)$$

$$v(n) = \sum_{-\infty}^{\infty} h(k)x(n-k)$$

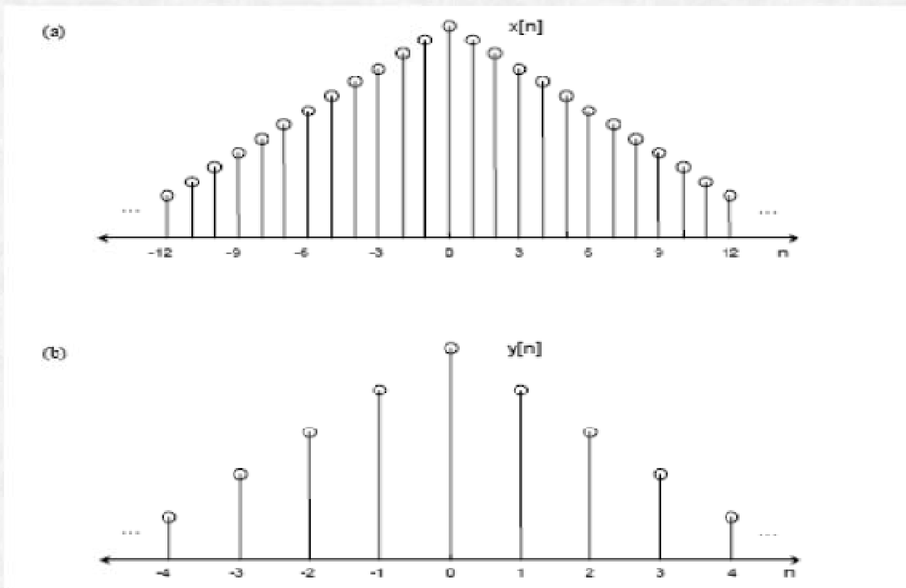
In decimation, the sampling rate is reduced from F_x to F_x/D by discarding D– 1 samples for every D samples in the original sequence

$$H_D = \begin{cases} 1, & |W| \le \pi/D \\ 0 & \text{otherwise} \end{cases}$$



Decimation by a factor *D*

M=3



The frequency domain representation of downsampling can be found by taking the z-transform to both sides of (1.5) as

$$Y(e^{j\omega T}) = \sum_{-\infty}^{+\infty} x(mM)e^{-j\omega Tm} = \frac{1}{M}\sum_{k=0}^{M-1} X(e^{j(\omega T - 2\pi k)/M}). \qquad (1.6)$$

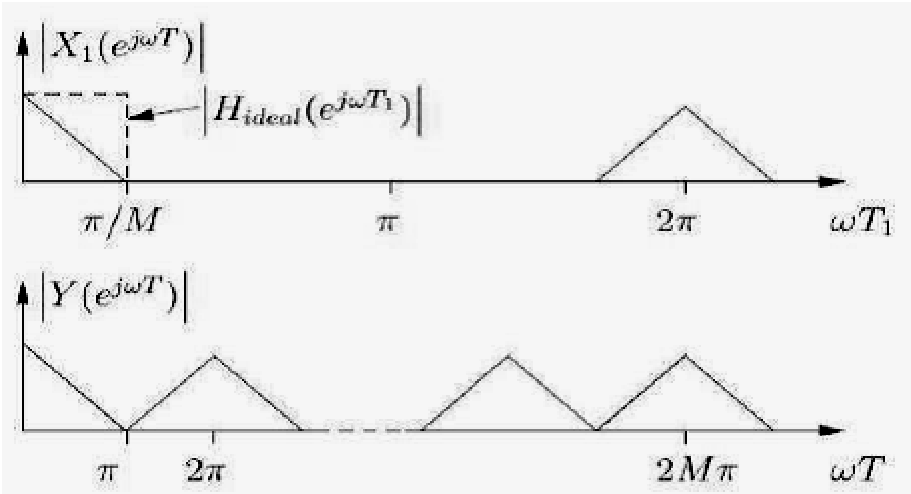


Fig. Spectra of the intermediate and decimated sequence

Example 8.2

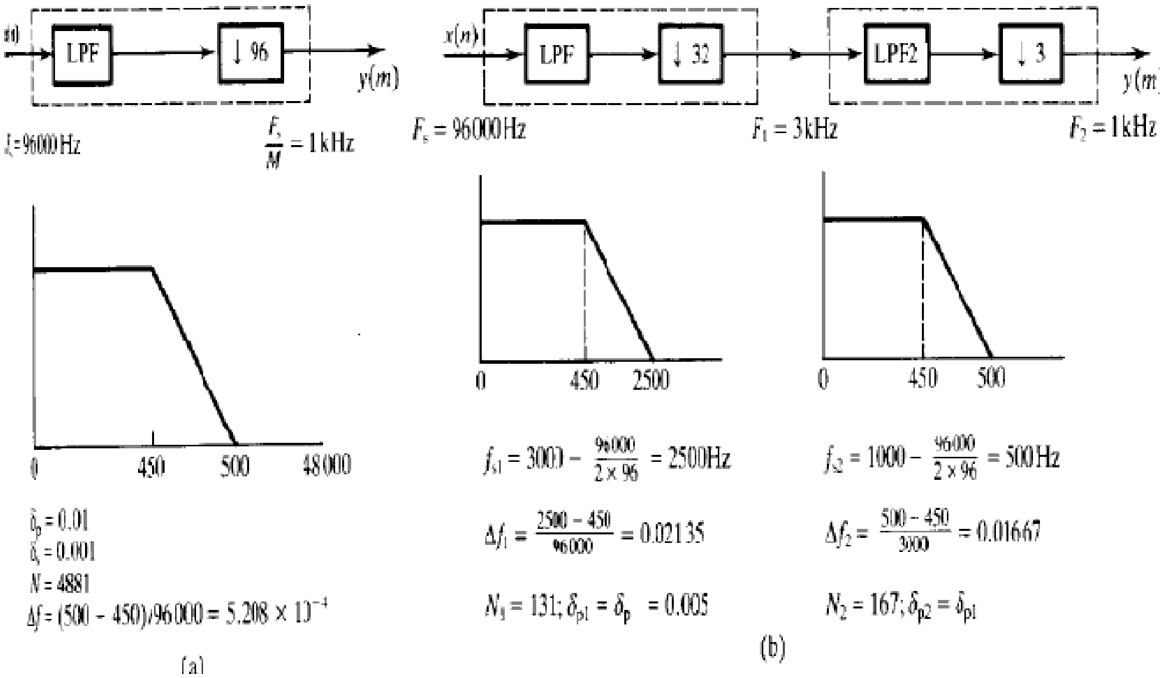
The sampling rate of a signal $x(n)$ is to be reduced, by decimation, from 96 kHz to 1 kHz. The highest frequency of interest after decimation is 450 Hz. Assume that an optimal FIR filter is to be used, with an overall passband ripple, $\delta_p = 0.01$, and passband deviation, $\delta_s = 0.001$. Design an efficient decimator.

Solution

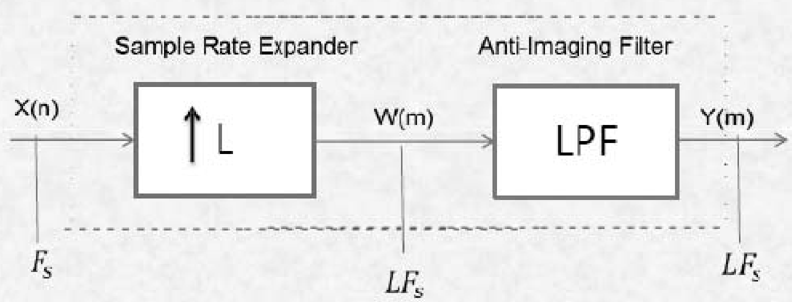
We will start by finding the most efficient design for each value of I, $I = 1, 2, 3, 4$. We will then compare these designs and select the best.

(1) First let us consider a one-stage design ($I = 1$). The block diagram and filter specifications for the stage are given in Figure 8.10(a).

(2) Next, we consider a two-stage design. Using the design program referred to in the text, the optimum integer decimation factors for $I = 2$ are $M_1 = 32$, $M_2 = 3$. The two-stage system, including its specifications, is shown in Figure 8.10(b). At the first stage, the sampling rate is reduced by 32 to 3 kHz, and, at the second stage, this is further reduced by 3 to 1 kHz.

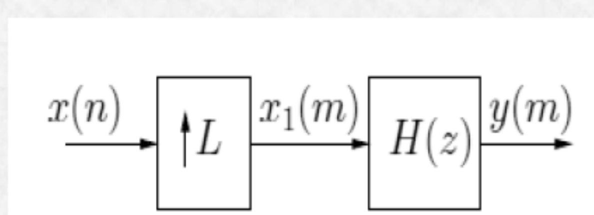


- Interpolation by a factor of L, where L is a positive integer, can be realized as a two-step process of upsampling followed by an anti-imaging filtering.



- An upsampling operation to a discrete-time signal x(n) produces an upsampled signal y(m) according to

$$y(m) = \begin{cases} x\left(\frac{n}{L}\right), & n = 0, \pm L, \pm 2L, \ldots, \\ 0, & \text{otherwise} \end{cases}$$



- The frequency domain representation of upsampling can be found by taking the z-transform of both sides

$$Y(e^{j\omega T}) = \sum_{-\infty}^{+\infty} y(m) e^{-j\omega T m} = X(e^{j\omega T L}).$$

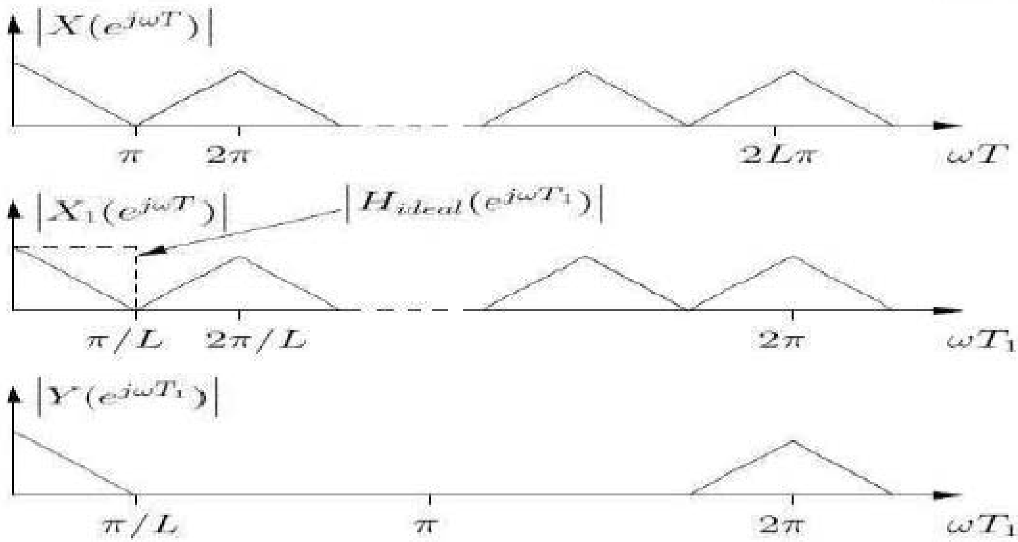


Fig. Spectra of the Original, intermediate and other sequences

Example 8.3

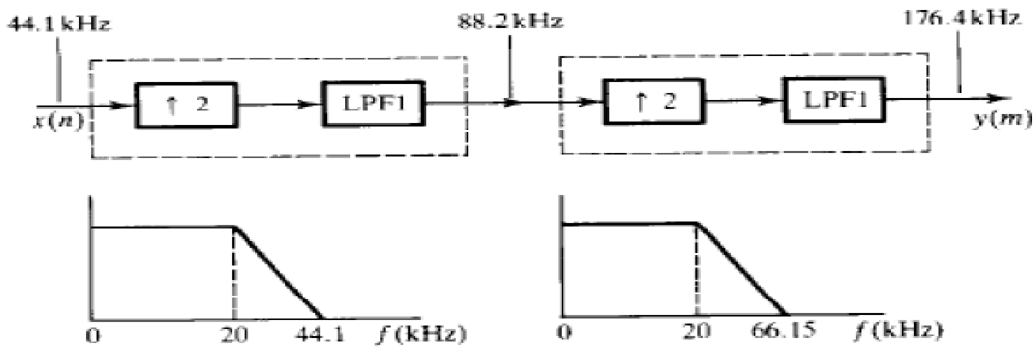
A digital audio system exploits oversampling techniques to relax the requirements of the analogue anti-imaging filter. The overall filter specifications for the system is given below:

baseband	0 to 20 kHz
input sampling frequency F_s	44.1 kHz
output sampling frequency	176.4 kHz
stopband attenuation	50 dB
passband ripple	0.5 dB
transition width	2 kHz
stopband edge frequency	22.05 kHz

Design a suitable interpolator.

Solution

Using the multirate design program on the PC disk for the book the interpolation factors and filter characteristics of the possible interpolators (with integer factors) are summarized below.



$\Delta f_1 = \frac{44.1 - 20}{88.2} = 0.02324$ $\qquad$ $\Delta f_1 = \frac{66.15 - 20}{176.4} = 0.26162$

$\delta_{P2} = 0.0296;\ \delta_{S2} = 0.00316$ $\qquad$ $\delta_{P1} = 0.0296;\ \delta_{S1} = 0.00316$

$N_2 = 83$ $\qquad$ $N_1 = 6$

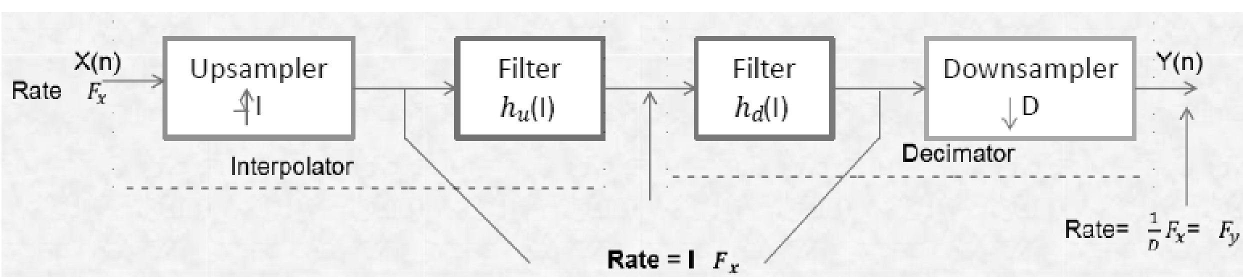
Figure 8.20 A two-stage interpolator for Example 8.3.

Number of stages	*Interpolation factor, L_i*	*Filter length N_i*	*Normalized transition width Δf_i*	*Passband ripple, δ_p*	*Stopband ripple, δ_s*
1	4	146	0.045 35	0.059 25	0.003 16
2	2	6	0.261 62	0.0296	0.003 16
	2	83	0.023 24	0.0296	0.003 16

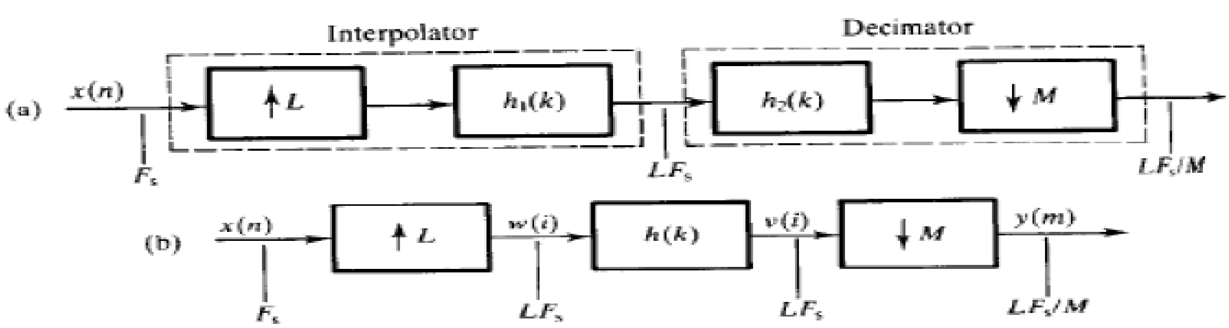
Sampling Rate conversion by Integer Rational Factor L/D

Sampling rate conversion by a rational factor 'L/D' can be achieved by first performing interpolation by the factor 'L' and then decimation the interpolator o/p by a factor 'D' .

In this process both the interpolation and decimator are cascaded as shown in the figure below:



Method for sampling rate conversion by a factor I/D.



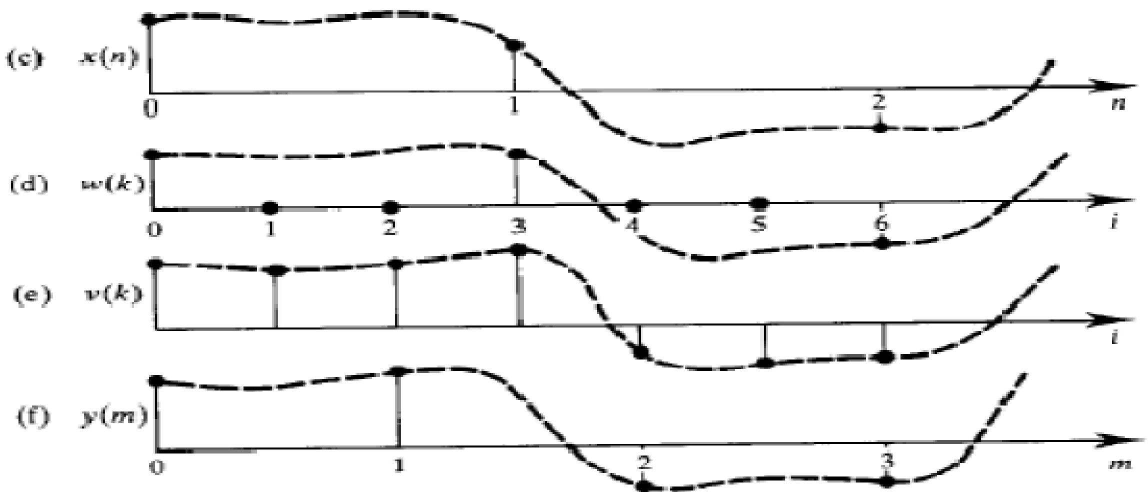


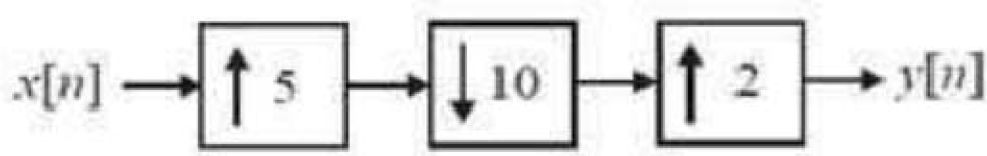
Fig. An illustration of interpolation by a rational factor (L=3, M=2)

- Example:

Consider a multirate signal processing problem:

i. State with the aid of block diagrams the process of changing sampling rate by a non-integer factor.

ii. Develop an expression for the output y[n] and g[n] as a function of input x[n] for the multirate structure of fig .



- Answer:

i. .

1. We perform the upsampling process by a factor L following of an interpolation filter h1(l).

2. We continue filtering the output from the interpolation filter via anti-aliasing filter h2(l) and finally operate downsampling.

ii.

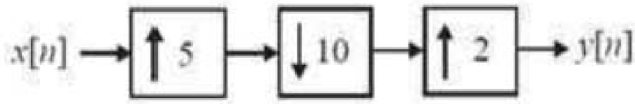
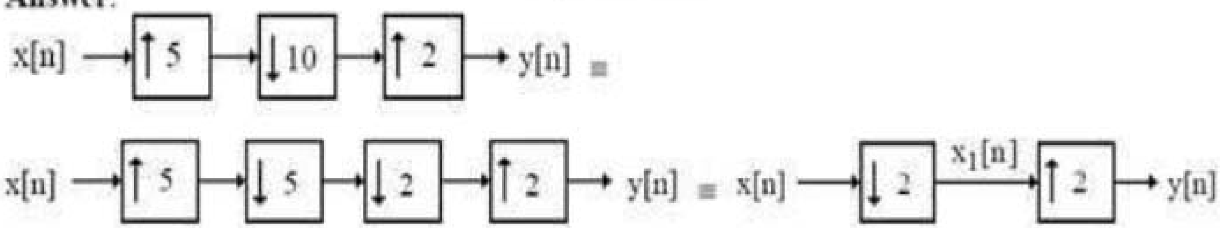


Figure E13.1

Answer:



Hence, $x_1[n] = x[2n]$ and $y[n] = \begin{cases} x_1[n/2], & \text{for } n = 2r, \\ 0, & \text{otherwise} \end{cases} = \begin{cases} x[n], & \text{for } n = 2r, \\ 0, & \text{otherwise} \end{cases}$ Therefore,

$$y[n] = \begin{cases} x[n], & \text{for } n = 2r, \\ 0, & \text{otherwise.} \end{cases}$$

Polyphase filters

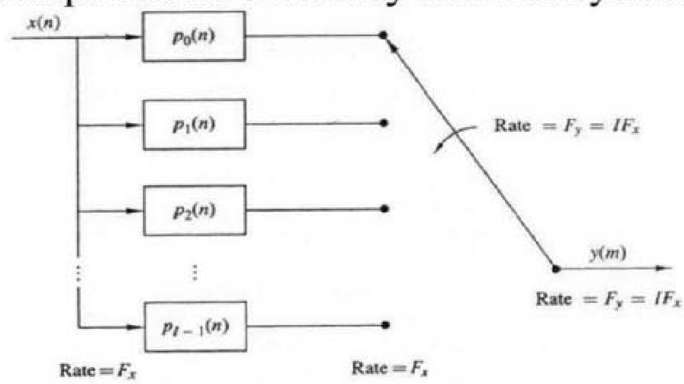
- Polyphase filters A very useful tool in multirate signal processing is the so-called poly phase representation of signals and systems facilitates considerable simplifications of theoretical results as well as efficient implementation of multirate systems.

- To formally define it, an LTI system is considered with a transfer function

$$H(z) = \sum_{n=-\infty}^{+\infty} h(n)z^{-n}.$$

Design and Implementation of Poly Phase Filter Structures for Sampling Rate Conversion

- The sampling rate conversion which is 'interpolation' ('decimation') is also performed by means of poly phase filter structures as shown in the figure below which results in better computational efficiency than FIR systems.



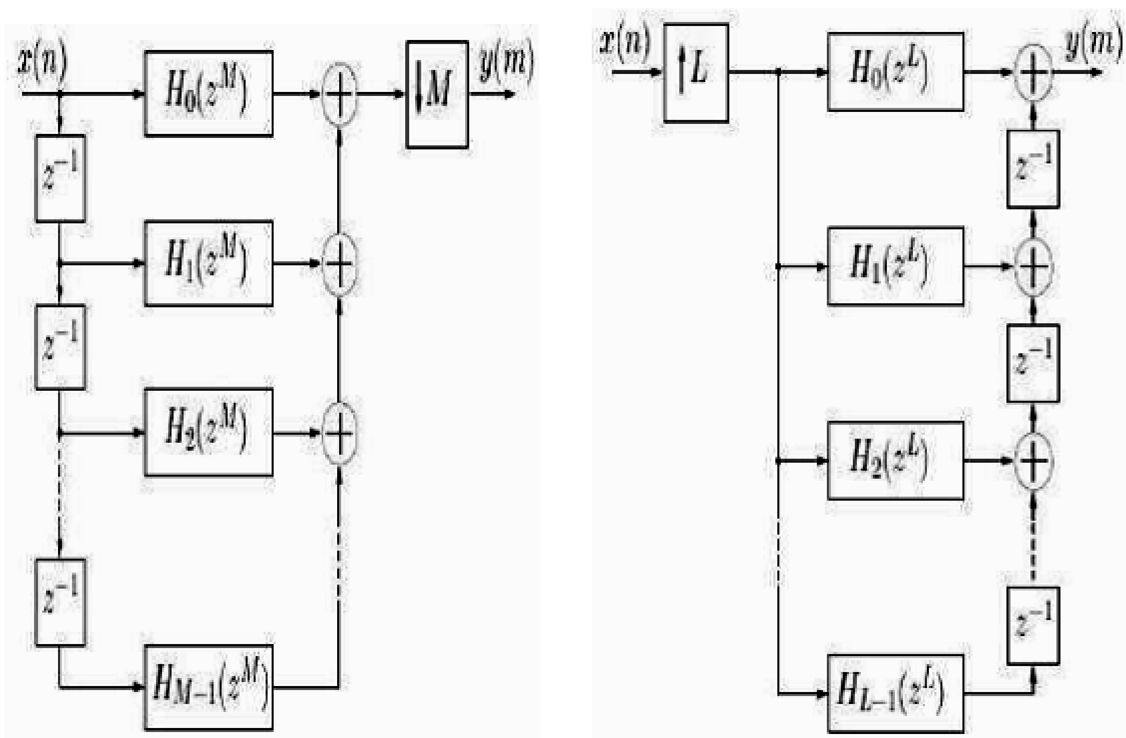


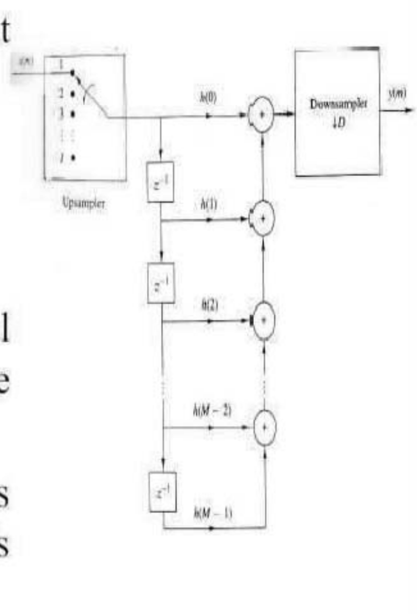
Fig. Polyphase decomposition of (a) a decimation filter (b) an interpolation filter

- This realization is simple but inefficient because,

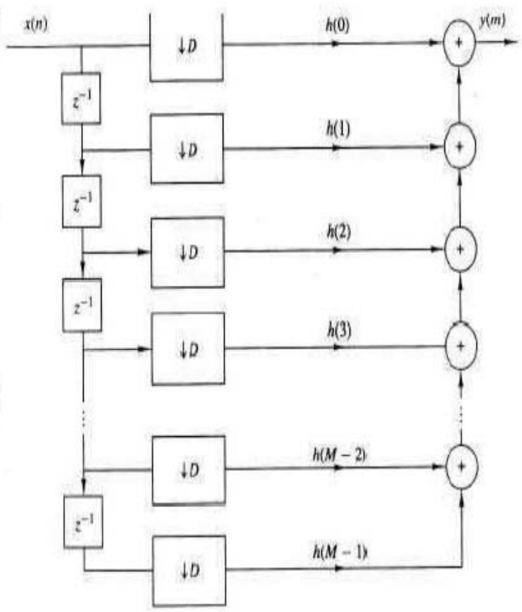
  1.up sampling process introduces *I-1* zero's between successive points of the signal.

  2.If '*I*' is large, most of the signal components in the FIR filter are zero.

  3.The multiplications and additions in the FIR filter result in zero's due to this large '*I*'.



- Therefore it is necessary to develop a more efficient structure.
- This can be achieved by embedding the down sampling operation within the filter it self as shown in the figure.



POLYPHASE FILTERING EXAMPLE (1)

- Consider Kth-order FIR filter with transfer function H given by coefficients b:

$$y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k]$$

- Example: downsampling by 3 after filtering, how to implement efficiently?

- Consider outputs after downsampling and rewrite by grouping coefficients with offsets of 3:

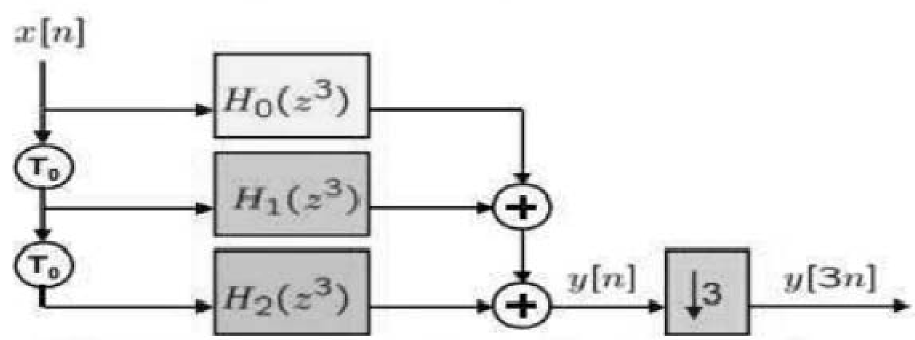
$$y[3n] = \sum_{k=0}^{K} b[k] \cdot x[3n-k]$$
$$= \sum_{k=0}^{K_0} b[3k] \cdot x[3(n-k)] + \sum_{k=0}^{K_1} b[3k+1] \cdot x[3(n-k)-1] + \sum_{k=0}^{K_2} b[3k+2] \cdot x[3(n-k)-2]$$

FIR filter with coefficients b[3k] applied to x[3n] — $H_0(z^3)$

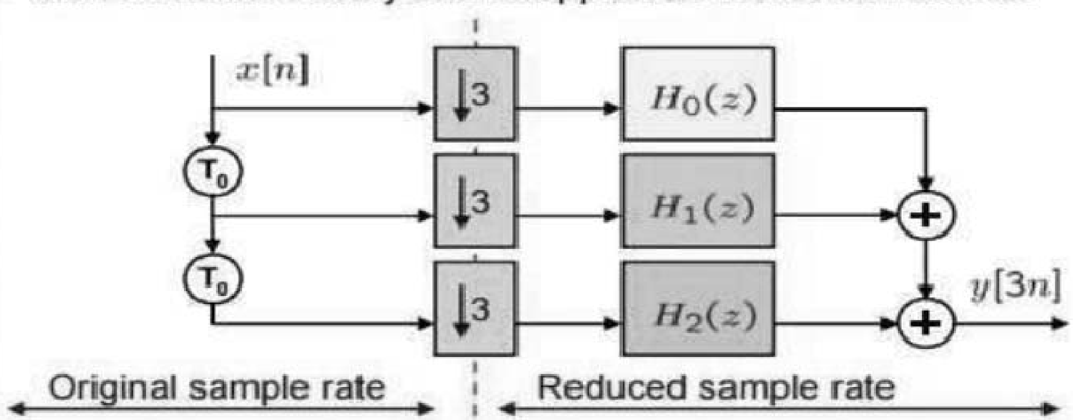
FIR filter with coefficients b[3k+1], applied to delayed x — $H_1(z^3)$

FIR filter with coefficients b[3k+2], applied to x delayed twice — $H_2(z^3)$

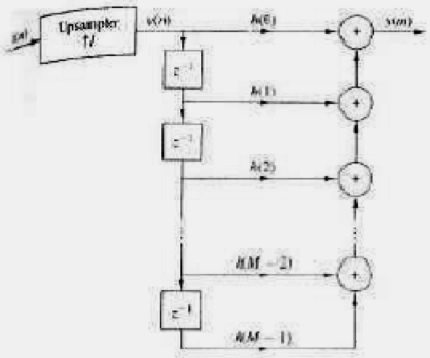
- Graphical representation of rewriting:

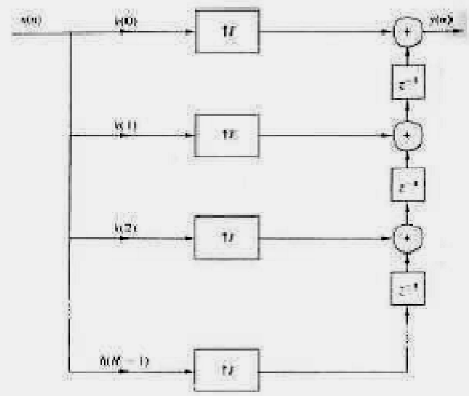


- Now the noble identity can be applied to the three subfilters:



- The major problem in this realization is that the filter computations are performed at high sampling rate ***Ifx***.
- This problem is solved by using transposed form of FIR filter and embedding the up sampler within the filter as shown in the figure.
- So all multiplications are performed at the lower rate ***Fx***.

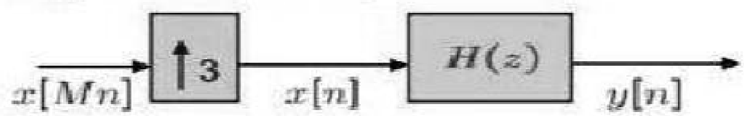




- Consider Kth-order **FIR** filter with transfer function ***H*** given b coefficients *b*:

$$y[n] = \sum_{k=0}^{K} b[k] \cdot x[n-k]$$

- Example: upsampling by 3 followed by filtering, how to implement efficiently?



- Start with definition, and group by coefficient index:

$$\begin{aligned} y[n] &= \sum_{k=0}^{K} b[k] \cdot x[n-k] \\ &= \sum_{k=0}^{K_0} b[3k] \cdot x[n-3k] + \\ &\quad \sum_{k=0}^{K_1} b[3k+1] \cdot x[n-3k-1] + \\ &\quad \sum_{k=0}^{K_2} b[3k+2] \cdot x[n-3k-2] \end{aligned}$$

Depending on *n*, only one out of three groups will be unequal to zero!

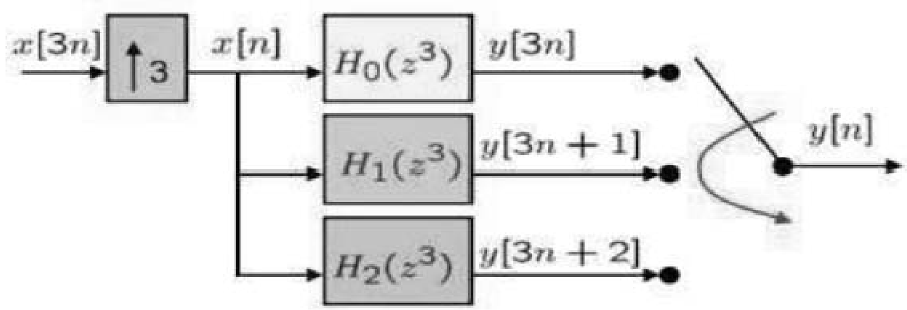
- Now consider outputs with different offsets separately and keep only those inputs unequal to zero.
- The result consists of three sequences that are filtered versions of the signal before upsampling.

$$y[3n] = \sum_{k=0}^{K_0} b[3k] \cdot x[3(n-k)] \qquad \boxed{H_0(z^3)}$$

$$y[3n+1] = \sum_{k=0}^{K_1} b[3k+1] \cdot x[3(n-k)] \qquad \boxed{H_1(z^3)}$$

$$y[3n+2] = \sum_{k=0}^{K_2} b[3k+2] \cdot x[3(n-k)] \qquad \boxed{H_2(z^3)}$$

- The previous equations represent:



Applications of Multirate DSP

- Multirate systems are used in a CD player when the music signal is converted from digital into analog (DAC).

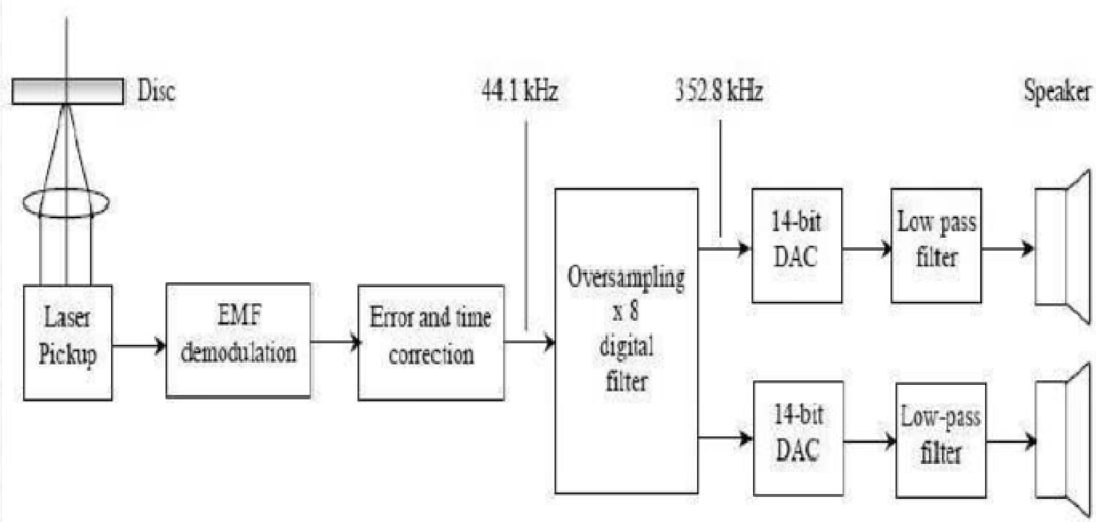
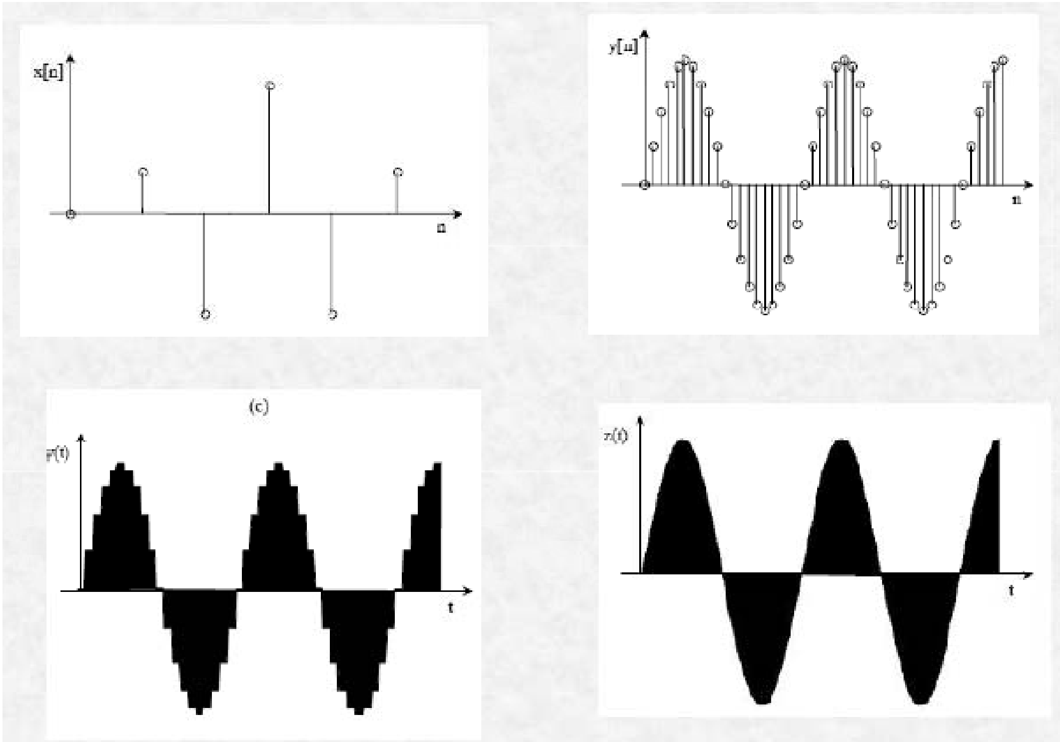


Fig. Digital to Analogue conversion for a CD player using x8 oversampling



The effect of oversampling also has some other desirable features:

Firstly, it causes the image frequencies to be much higher and therefore easier to filter out.

Secondly reducing the noise power spectral density, by spreading the noise power over a larger bandwidth.

$$Noise\ \ power\ \ spectral\ \ density = \frac{Total\ \ power}{Bandwidth}$$

High quality Analog to Digital conversion for digital audio

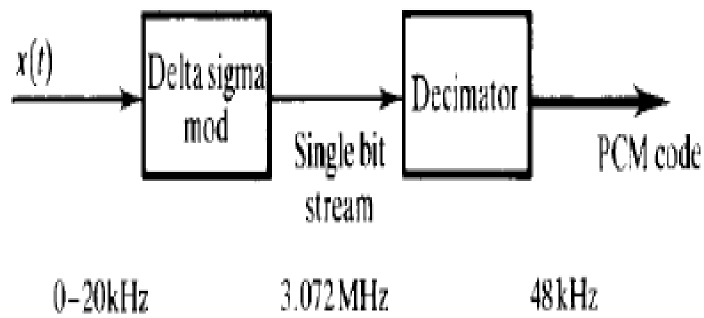


Fig. Simplified block diagram of single-bit ADC scheme

Speech Compression

Data Rates

- Telephone quality voice:
  - 8000 samples/sec, 8 bits/sample, mono
  - 64Kb/s
- CD quality audio:
  - 44100 samples/sec, 16 bits/sample, stereo
  - ~1.4Mb/s
- Communications channels and storage cost money (although less than they used to)
  - What can we do to reduce the transmission and/or storage costs without sacrificing too much quality?

Speech Codec Overview

- PCM - send every sample
- DPCM - send differences between samples
- ADPCM - send differences, but adapt how we code them
- SB-ADPCM - wideband codec, use ADPCM twice, once for lower frequencies, again at lower bitrate for upper frequencies.
- LPC - linear model of speech formation
- CELP - use LPC as base, but also use some bits to code corrections for the things LPC gets wrong.

PCM

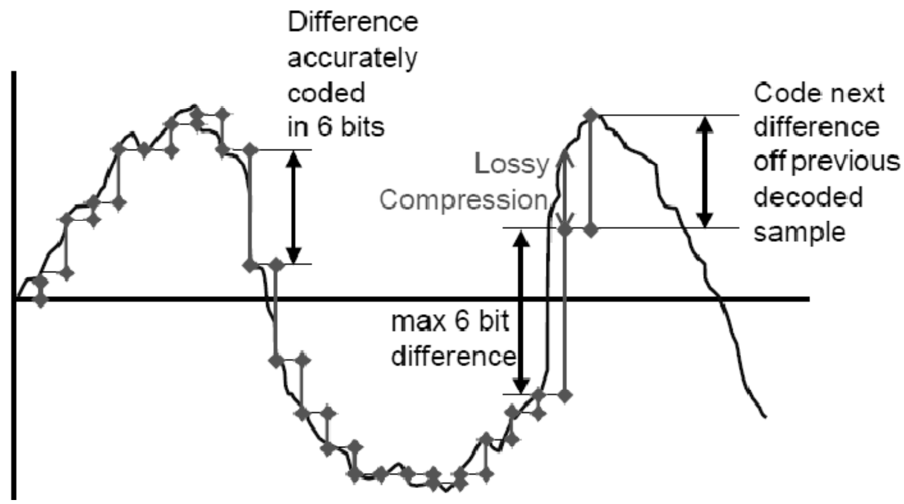
- μ-law and a-law PCM have already reduced the data sent.
- Lost frequencies above 4KHz.
- Non-linear encoding to reduce bits per sample.

- However, each sample is still independently encoded.
  - In reality, samples are correlated.
  - Can utilize this correlation to reduce the data sent.

Differential PCM

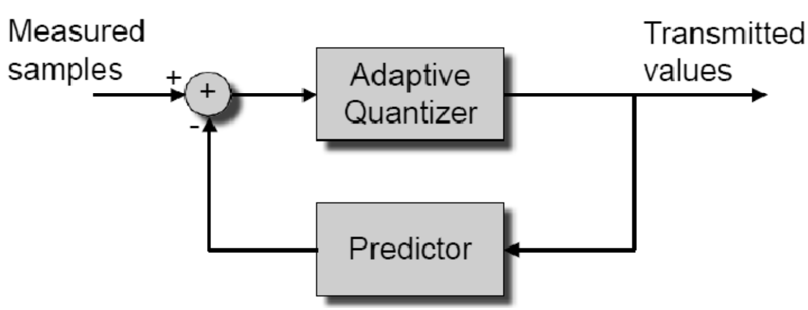
- Normally the difference between samples is relatively small and can be coded with less than 8 bits.
- Simplest codec sends only the differences between samples.
  - Typically use 6 bits for difference, rather than 8 bits for absolute value.

- Compression is *lossy, as* not all differences can be coded
  - Decoded signal is slightly degraded.
  - Next difference must then be encoded off the previous



ADPCM (Adaptive Differential PCM)

- Makes a simple prediction of the next sample, based on weighted previous *n* samples.
  - For G.721, previous 8 weighted samples are added to make the prediction.
- Lossy coding of the difference between the actual sample and the prediction.
  - Difference is quantized into 4 bits ⇒ 32Kb/s sent.
  - Quantization levels are adaptive, based on the content of the audio.
- Receiver runs same prediction algorithm and adaptive quantization levels to reconstruct speech.



- Adaptive quantization cannot always exactly encode a difference.
  - Shows up as quantization noise.
- Modems and fax machines try to use the full channel capacity.
  - If they succeed, one sample is not predictable from the next.
  - ADPCM will cause them to fail or work poorly.

- ADPCM not normally used on national voice circuits, but commonly used internationally to save capacity on expensive satellite or undersea fibres.

Predictor Error

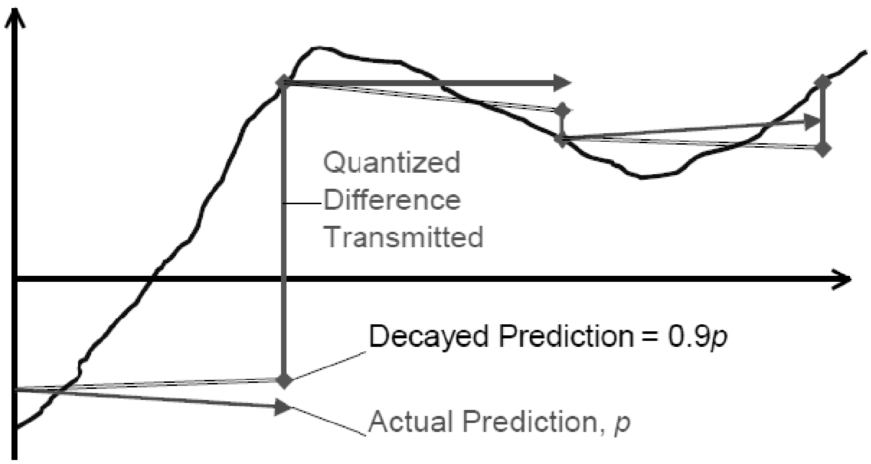
- What happens if the signal gets corrupted while being transmitted?
  - Wrong value will be decoded.
  - Predictor will be incorrect.
  - All future values will be decoded incorrectly!

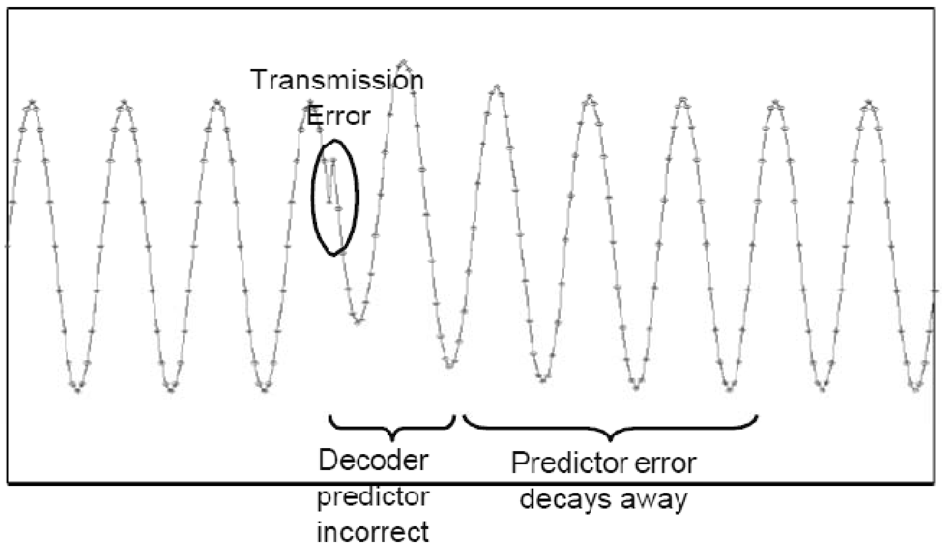
- Modern voice circuits have low but non-zero error rates.
  - But ADPCM was used on older circuits with higher loss rates too. How?

ADPCM Predictor Error

- Want to design a codec so that errors do not persist.
- Build in an automatic decay towards zero.
  - If only differences of zero were sent, the predictor would decay the predicted (and hence decoded) value towards zero.

- Differences have a mean value of zero (there are as many positive differences as negative ones).



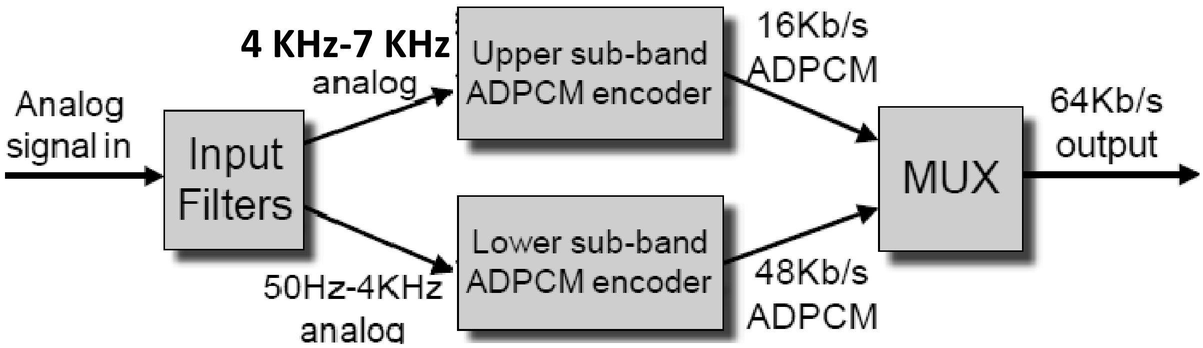


Sub-band ADPCM

- Regular ADPCM reduces the bitrate of 8KHz sampled audio (typically 32Kb/s).
- If we have a 64Kb/s channel (eg ISDN), we could use the same techniques to produce better that toll-quality.
- Could just use ADPCM with 16KHz sampled audio, but not all frequencies are of equal importance.
  - 0-3.5KHz important for intelligibility
  - 3.5-7KHz helps speaker recognition and conveys emotion

Sub-band ADPCM

Filter into two bands:
50 Hz-3.5 KHz: sample at 8 kHz, encode at 48 KB/s
3.5 KHz- 7 KHz: sample at 16 kHz, encode at 16 KB/s



- Practical issue:
  - Unless you have dedicated hardware, probably can't sample two sub-bands separately at the same time.
  - Need to process digitally.
    - Sample at 16KHz.
    - Use digital filters to split sub-bands and downsample the lower sub-band to 8KHz.

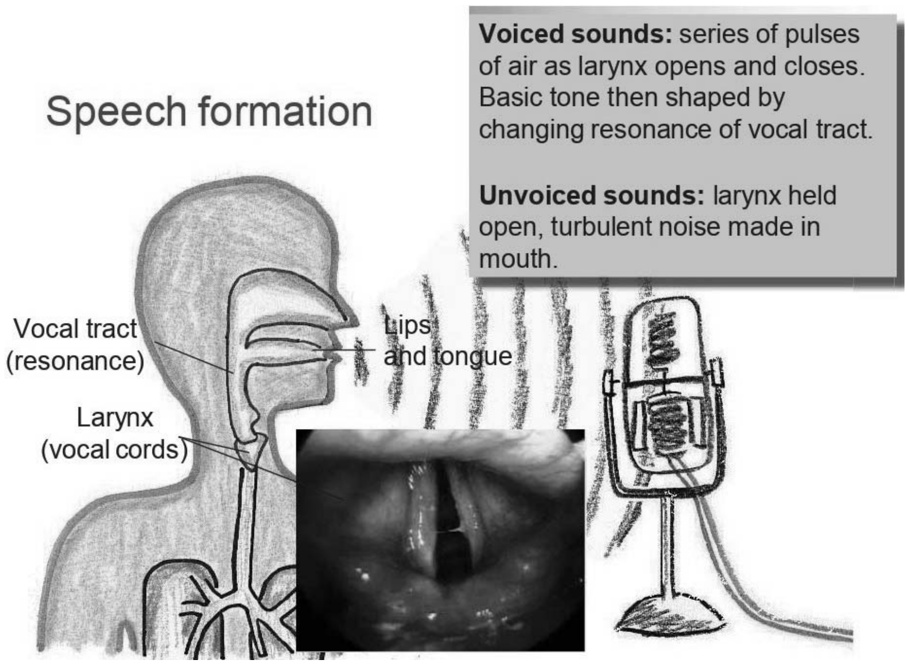
Key point of Sub-band ADPCM:

- Not all frequencies are of equal importance (quantization noise is more disruptive to some parts of the signal than others)
- Allocate the bits where they do most good.

Model-based Coding

- PCM, DPCM and ADPCM directly code the received audio signal.
- An alternative approach is to build a *parameterized model of the sound source* (ie. Human voice).

- For each time slice (eg 20ms):
  - Analyse the audio signal to determine how the signal was produced.
  - Determine the model parameters that fit.
  - Send the model parameters.
- At the receiver, synthesize the voice from the model and received parameters.

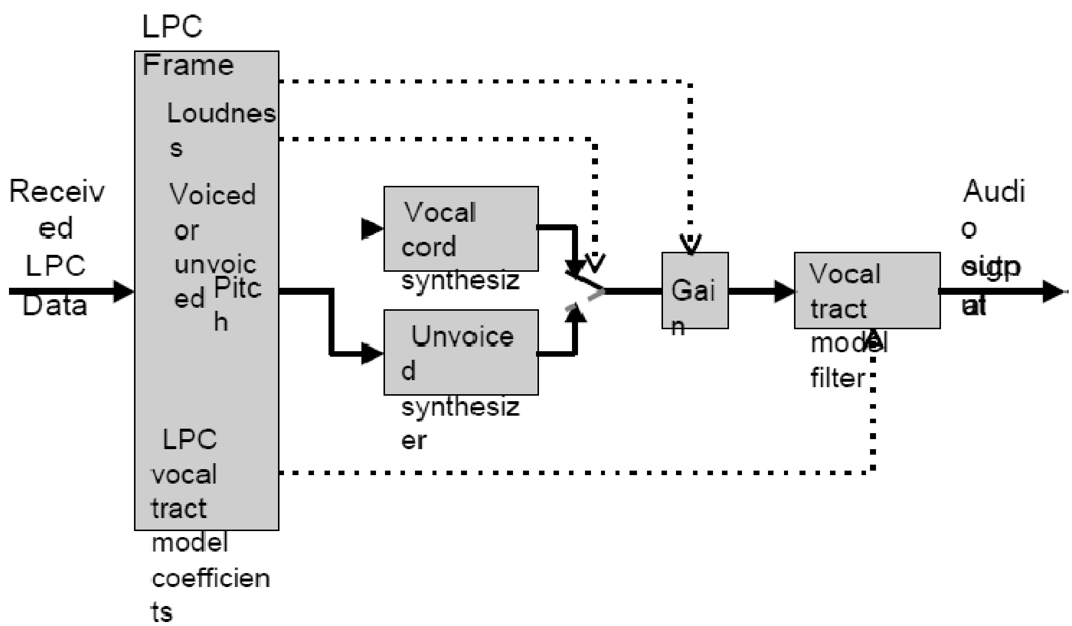


Linear Predictive Coding (LPC)

- Introduced in 1960s.
- Low-bitrate encoder:
  - 1.2Kb/s - 4Kb/s
- Sounds very synthetic
  - Basic LPC mostly used where bitrate really matters (eg in miltary applications)
  - Most modern voice codecs (eg GSM) are based on enhanced LPC encoders.

- Digitize signal, and split into segments (eg 20ms)
- For each segment, determine:
  - Pitch of the signal (ie basic formant frequency)
  - Loudness of the signal.
  - Whether sound is voiced or unvoiced
    - Voiced: vowels, “m”, “v”, “l”
    - Unvoiced: “f”, “s”
- Vocal tract excitation parameters (LPC Coefficients)

LPC Decoder



- Vocal chord synthesizer generates a series of impulses.
- Unvoiced synthesizer is a white noise source.

- Vocal tract model uses a linear predictive filter.
  - n^{th} sample is a linear combination of the previous p samples plus an error term:

$$x_n = a_1 x_{n-1} + a_2 x_{n-2} + .. + a_n x_{n-p} + e_n$$

  - e_n comes from the synthesizer.
  - The coefficients $a_1 .. a_p$ comprise the vocal tract model, and shape the synthesized sounds.

LPC Encoder

- Once pitch and voice/unvoiced are determined, encoding consists of deriving the optimal LPC coefficients ($a_1 .. a_p$) for the vocal tract model so as to minimize the mean-square error between the predicted signal and the actual signal.
- Problem is straightforward in principle. In practice it involves:
  1. The computation of a matrix of coefficient values.
  2. The solution of a set of linear equations.
  - Several different ways exist to do this efficiently (autocorrelation, covariance, recursive latice formulation) to assure convergence to a unique solution.

Limitations of LPC Model

- LPC linear predictor is very simple.
  - For this to work, the vocal tract "tube" must not have any side branches (these would require a more complex model).
  - OK for vowels (tube is a reasonable model)
  - For nasal sounds, nose cavity forms a side branch.
- In practice this is ignored in pure LPC.
  - More complex codecs attempt to code the residue signal, which helps correct this.

Code Excited Linear Prediction (CELP)

- Goal is to efficiently encode the residue signal, improving speech quality over LPC, but without increasing the bit rate too much.
- CELP codecs use a codebook of typical residue values.
  - Analyzer compares residue to codebook values.
  - Chooses value which is closest.
  - Sends that value.
- Receiver looks up the code in its codebook, retrieves the residue, and uses this to excite

- Problem is that codebook would require different residue values for every possible voice pitch.
  - Codebook search would be slow, and code would require a lot of bits to send.
- One solution is to have two codebooks.
  - One fixed by codec designers, just large enough to represent one pitch period of residue.
  - One dynamically filled in with copies of the previous residue delayed by various amounts (delay provides the pitch)
- CELP algorithm using these techniques can provide pretty good quality at 4.8Kb/s.

Enhanced LPC Usage

- GSM (Groupe Speciale Mobile)
  - Residual Pulse Excited LPC
  - 13Kb/s
- LD-CELP
  - Low-delay Code-Excited Linear Prediction (G.728)
  - 16Kb/s
- CS-ACELP
  - Conjugate Structure Algebraic CELP (G.729)
  - 8Kb/s
- MP-MLQ

Adaptive Filters

- **the signal and/or noise characteristics are often *nonstationary* and the statistical parameters vary with time**

- **An adaptive filter has an *adaptation algorithm*, that is meant to monitor the environment and vary the filter transfer function accordingly**

- **based in the actual signals received, attempts to find the optimum filter design**

The basic operation now involves two processes :

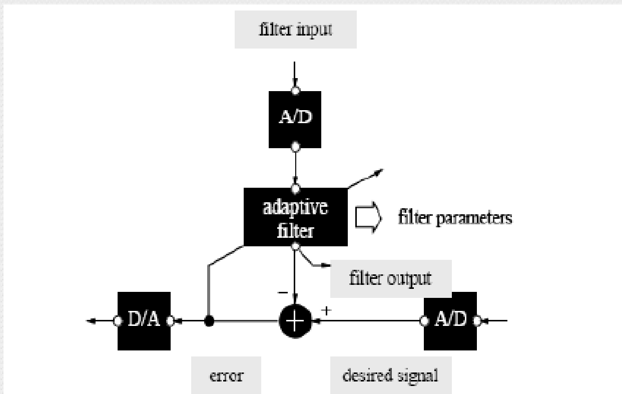
1. a *filtering* process, which produces an output signal in response to a given input signal.

2. an adaptation process, which aims to adjust the filter parameters (filter transfer function) to the (possibly time-varying) environment

Often, the (average) square value of the error signal is used as the optimization criterion

- Because of complexity of the optimizing algorithms most adaptive filters are digital filters that perform digital signal processing

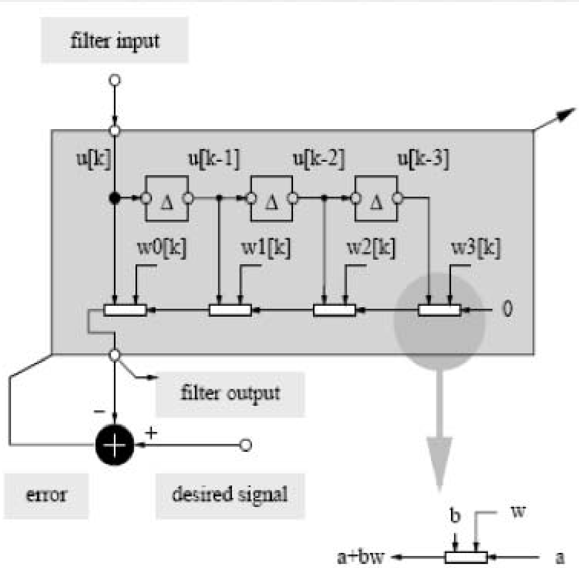
When processing analog signals, the adaptive filter is then preceded by A/D and D/A convertors.



Prototype adaptive digital filtering scheme with A/D and D/A

- The generalization to adaptive IIR filters leads to stability problems

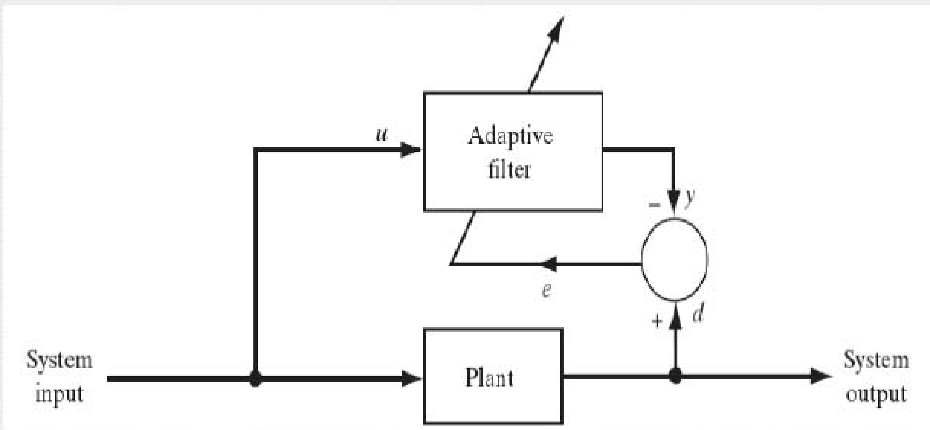
- It's common to use a FIR digital filter with adjustable coefficients



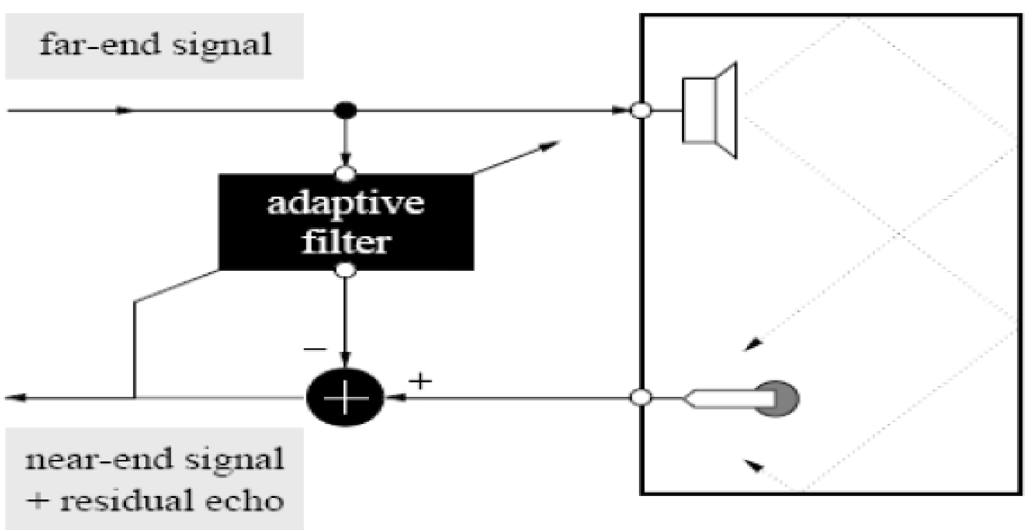
Prototype adaptive FIR filtering scheme

Adaptive Filters - Applications

- Used to provide a linear model of an unknown plant
- Applications:
  - System identification



Echo Cancellation



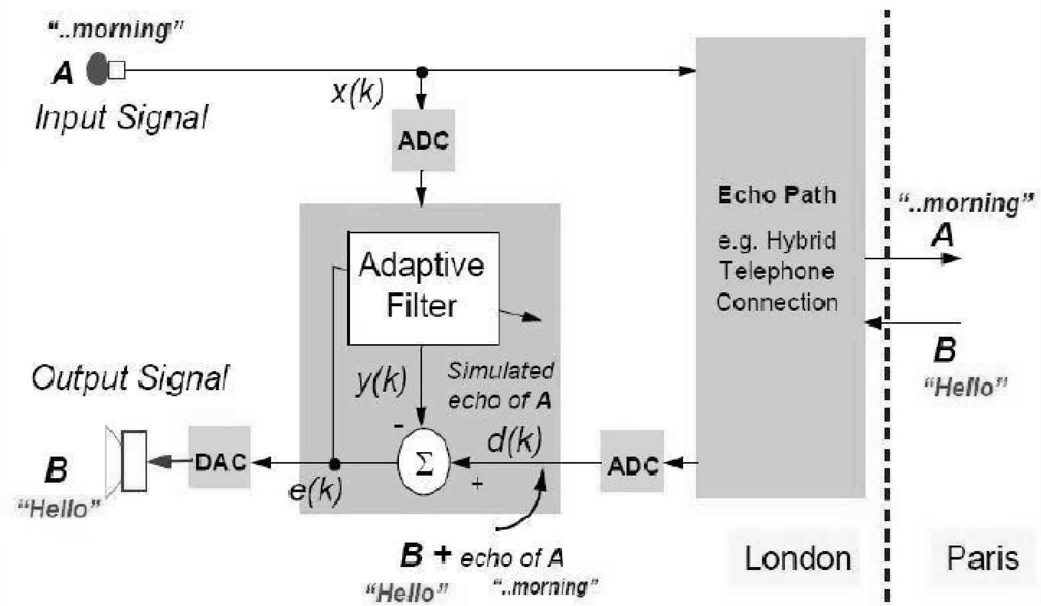
Acoustic echo cancellation

Application Examples

- **System Identification:**
    - Channel identification; Echo Cancellation
- **Inverse System Identification:**
    - Digital communications equalisation.
- **Noise Cancellation:**
    - Active Noise Cancellation; Interference cancellation for CDMA
- **Prediction:**
    - Periodic noise suppression; Periodic signal extraction; Speech coders; CMDA interference suppression.

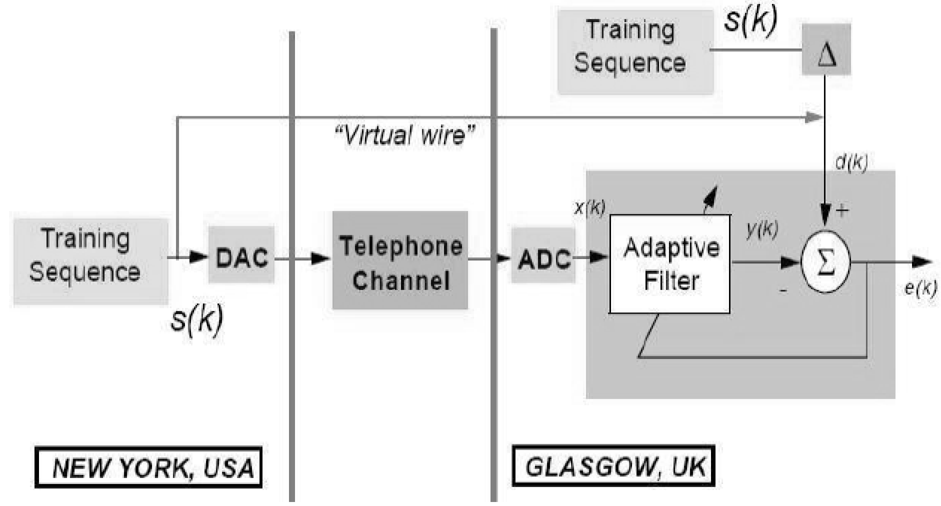
Echo Cancellation

- Local line echo cancellation is widely used in data modems (V-series) and in telephone exchanges for echo reduction.



Channel Equalisation

- To improve the bandwidth of a channel we can attempt to equalise a communication channel:



Applications are many

- Digital Communications (OFDM , MIMO , CDMA, and RFID)
- Channel Equalisation
- Adaptive noise cancellation
- Adaptive echo cancellation
- System identification
- Smart antenna systems
- Blind system equalisation
- And many, many others

New Trends in Adaptive Filtering

- **Partial Updating Weights.**
- **Sub-band adaptive filtering.**
- **Adaptive Kalman filtering.**
- **Affine Projection Method.**
- **Time-Space adaptive processing.**
- **Non-Linear adaptive filtering:-**
- **Neural Networks.**
- **The Volterra Series Algorithm .**
- **Genetic & Fuzzy.**
- **Blind Adaptive Filtering.**

Musical Sound Processing

The audio effects are artificially generated using various signal processing circuits and devices, and increasingly by digital signal processing techniques, often referred as Musical Sound processing. All musical programs are produced in basically two stages:

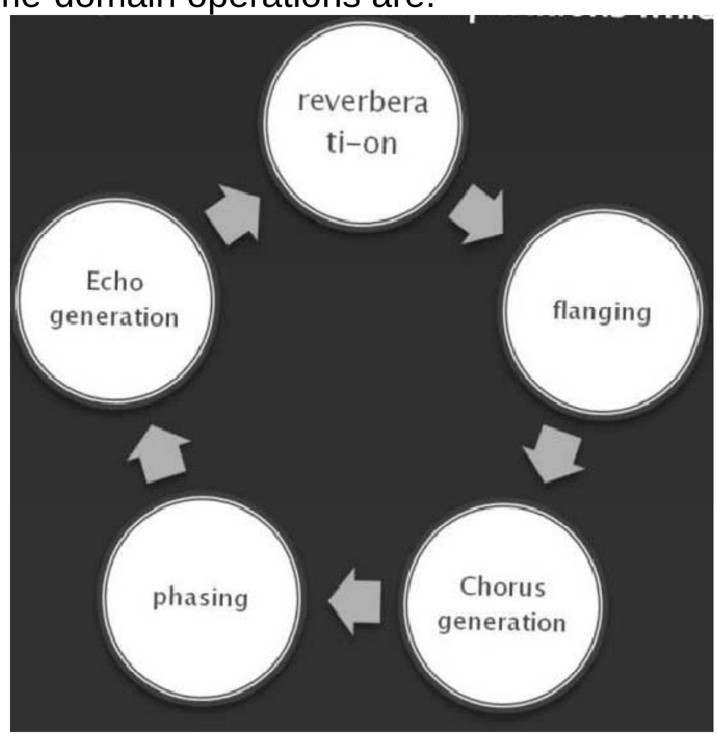
Sound from each individual instrument is recorded in an acoustically inert studio on a single track of a multi-track tape recorder

→

The signals from each track are manipulated by the sound engineer to add special audio effects and are combined in a mix-down system to finally generate the stereo recording on a two-track tape

Time Domain Operation

Commonly used time-domain operations are:

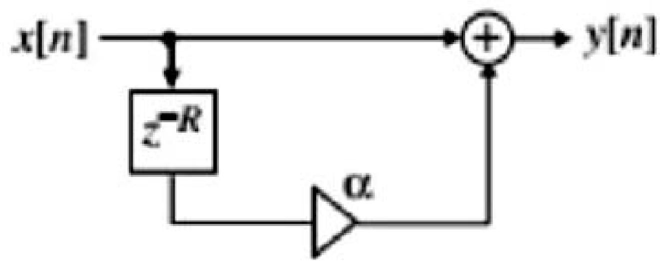


Single Echo Filter

Echo are simply generated by delay units. Because of the com like shape of the magnitude response, such a filter is known as comb filter.

For example, the direct sound and a single echo appearing R sampling periods later can be simply generated by the FIR filter shown in Fig., which is characterized by the difference equation:

$$y[n] = x[n] + \alpha\, x[n-R], \quad |\alpha| < 1$$

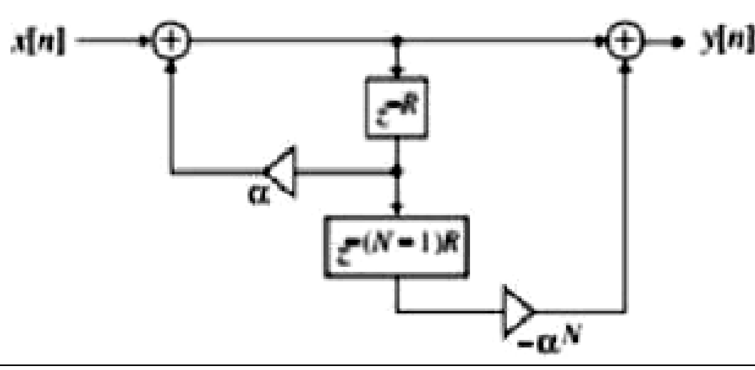


Multiple Echo Filter

To generate a fixed number of multiple echoes spaced R sampling periods with the exponentially decaying amplitudes.

One can use an FIR filter with a transfer function of the form:

$$H(z) = \frac{1 - \alpha^N z^{-NR}}{1 - \alpha z^{-R}}$$



Reverberation

- The reverberation is considered o densely packed echoes.

- The IIR comb filter by itself does not provide natural-sounding reverberation for two reasons, which are:

  - Its magnitude response is not constant for all frequencies, resulting in a "coloration" of many musical sound that are often unpleasant for listening.

  - The output echo density given by number of echoes per second generated by a unit impulse at the input is much lower than that observed in a real room thus causing "fluttering" of the composite sound.

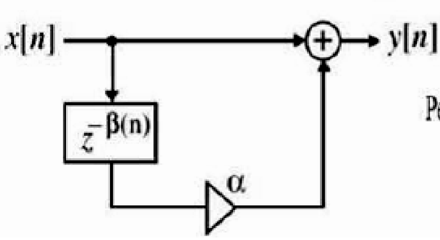
Flanging

- A number of special sound effects are often used in the mix-down process. One such effect is flanging.

- It was created by feeding the same musical piece to tape recorders and then combining their delayed outputs while varying the difference between their delay.

- One way of varying time is to slow down one of the tape recorders by placing the operators thumb on the flange of the feed reel, which is led to the name flanging.

- Flanging Effect:



$$y[n] = x[n] + \alpha x[n - \beta(n)]$$

Periodically varying the delay $\beta(n)$ between 0 and R with a low frequency ω_o such as

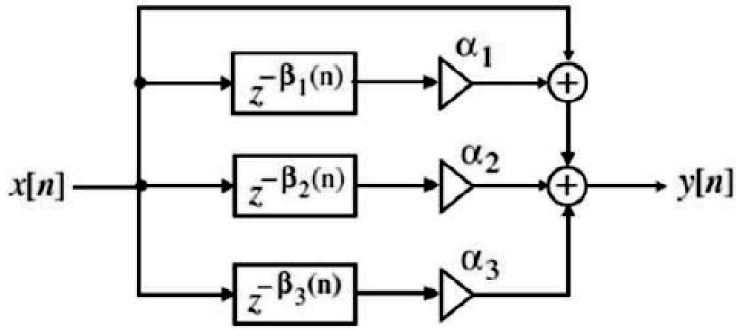
$$\beta(n) = \frac{R}{2}(1 - \cos(\omega_o n))$$

Chorus Generator and Phasing

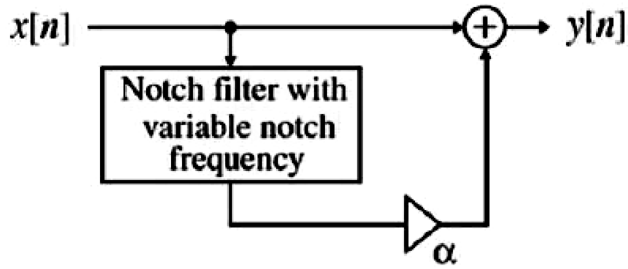
- The chorus effect is achieved when several musicians are playing the same musical piece at the same time but with small changes in the amplitude and small timing differences between their sounds.

- The phasing effect is produced by processing the signal through a narrowband notch filter with variable botch characteristics and adding a scaled portion of the notch filter output the original signal.

❖ Chorus Effect:



❖ Phasing Effect:



Frequency-Domain Operations

❖ These effects are achieved by passing the original signals through an equalizer, the purpose of equalizer is to provide "presence" by peaking the mid-frequency components in the range of 1.5 GHz to 3 GHz and to modify the bass-treble relationships by providing boost or cut to componnets outside this range.

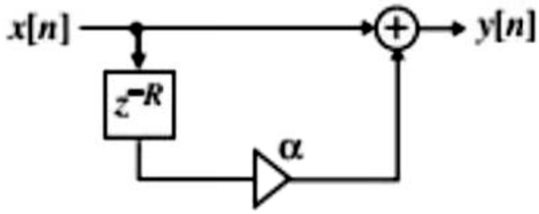
❖ Advantages:

- an information can be conveyed, displayed or manipulated
- perfect reproducibility-identical performance from unit to unit.
- Guaranteed accuracy is only determined by the number of bits used.
- Stored almost indefinitely without loss of information

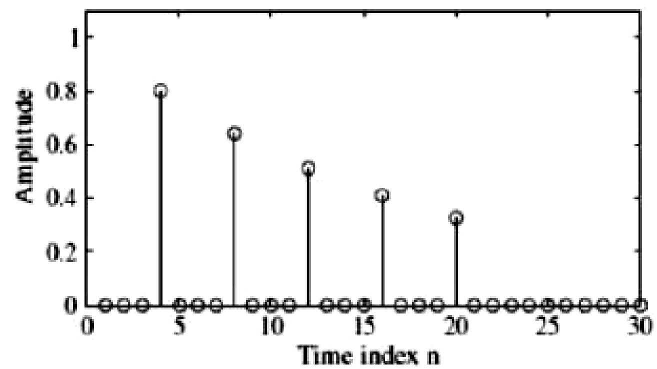
❖ Disadvantages:

- Speed and cost- be expensive with large bandwidth signals
- DSP designs can be time consuming plus need the necessary resources (software etc.)
- Finite word-length problems - if only a limited number of bits is used due to economic considerations, serious degradation may result in system performance.

Output from the Algorithm
Single Echo Filter

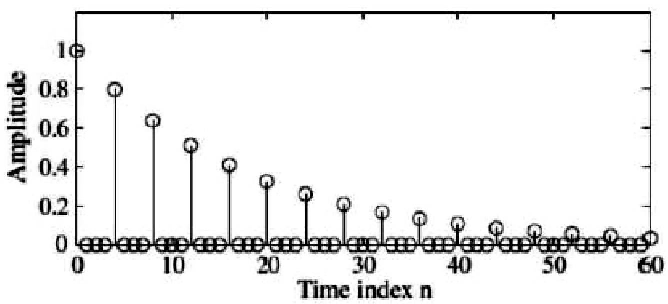


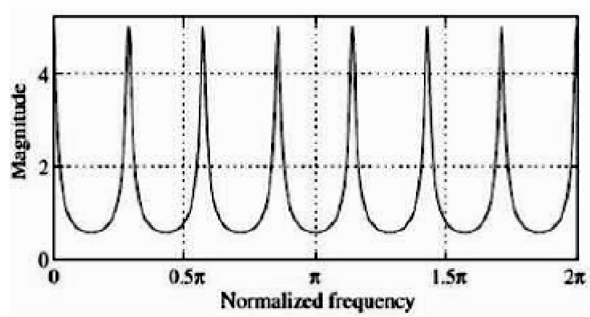
Filter Structure



Impulse Response with
A=0.8, N=6 & R=4

Multiple Echo Filter

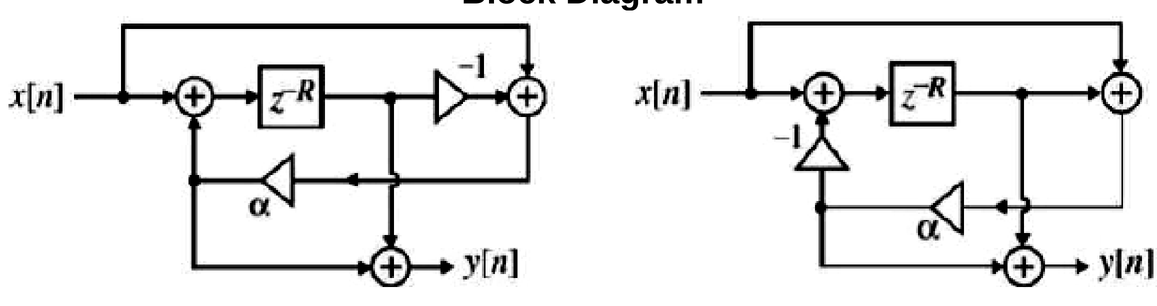




Impulse Response with A=0.8 for R=4 **Magnitude Response for R=7**

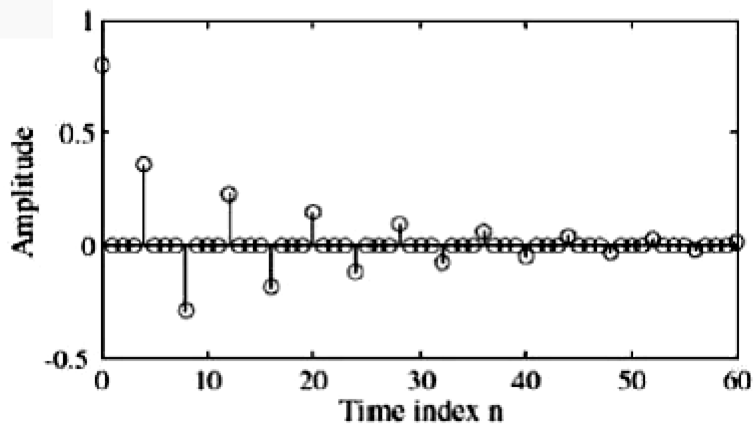
Reverberation

Block Diagram



The transfer function of the allpass reverberator is given by

$$H(z) = \frac{\alpha + z^{-R}}{1 + \alpha z^{-R}}, \quad |\alpha| < 1$$



Impulse Response with A=0.8 for R=4

Image Enhancement

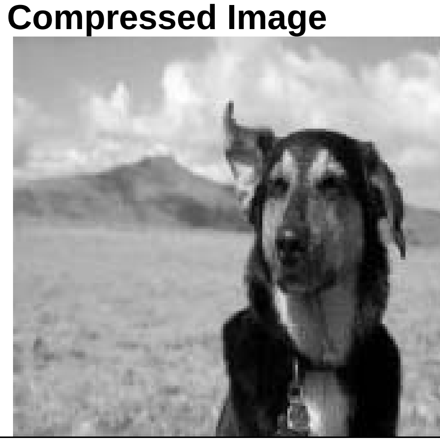
An **image** defined in the "real world" is considered to be a function of two real variables, for example, a(x,y) with a as the amplitude (e.g. brightness) of the image at the real coordinate position (x,y)

Image processing is the study of any algorithm that takes an image as input and returns an image as output. It includes the following:

1. Image display and printing
2. Image editing and manipulation
3. Image enhancement
4. Feature detection
5. Image compression.

Original Image

Compressed Image



WHY IMAGE ENHANCEMENT?

- The aim of image enhancement is to improve the visual appearance of an image, or to provide a “better transform representation for future automated image processing.
- Many images like medical images, satellite images, aerial images and even real life photographs suffer from poor contrast and noise.
- It is necessary to enhance the contrast and remove the noise to increase image quality.
- Enhancement techniques which improves the quality (clarity) of images for human viewing, removing blurring and noise, increasing contrast, and revealing details are examples of enhancement operations.

WHAT IS IMAGE ENHANCEMENT?

- **Image enhancement process** consists of a collection of techniques that seek to improve the visual appearance of an image or to convert the image to a form better suited for analysis by a human or machine.
- **The principal objective** of image enhancement is to modify attributes of an image to make it more suitable for a given task and a specific observer.

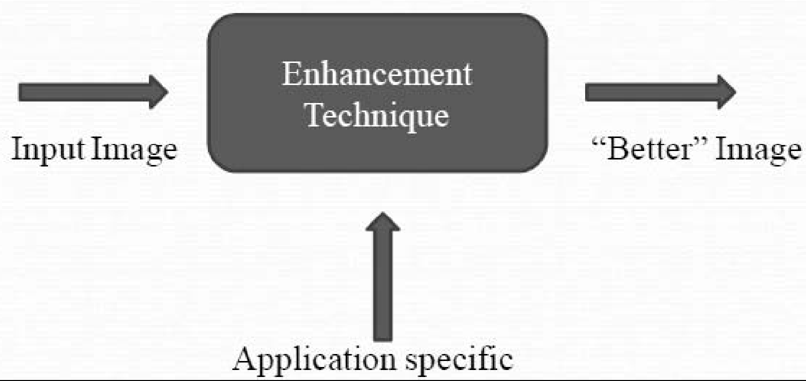


IMAGE ENHANCEMENT TECHNIQUES:

The existing techniques of image enhancement can be classified into two categories:

- Spatial domain enhancement
- Frequency domain enhancement.

Image Enhancement

Point operation
- Contrast stretching
- Noise clipping
- Window slicing
- Histogram modelling

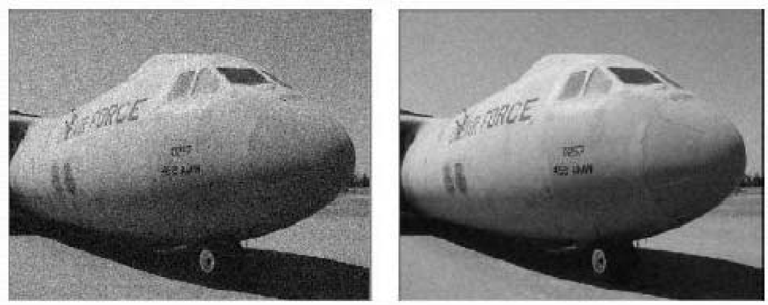
Spatial operation
- Noise smoothing
- Median filtering
- LP, HP & BP filtering
- Zooming

Transform Operation
- Linear filtering
- Root filtering
- Homomorphic filtering

Pseudo-coloring
- False coloring
- Pseudo-coloring

Examples

1. Noise removal



Noisy image De-noised image

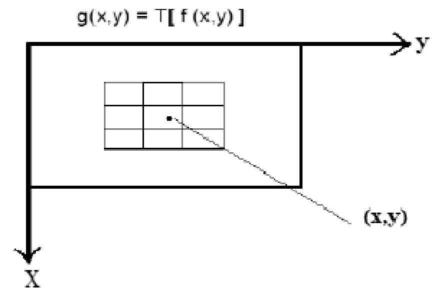
2. Contrast adjustment

Low contrast Original contrast High contrast

Spatial Domain Enhancement

•Spatial domain techniques are performed to the image plane itself and they are based on direct manipulation of pixels in an image.

• The operation can be formulated as $g(x,y)=T[f(x,y)]$, where g is the output, f is the input image and **T** is an operation on f defined over some neighbourhood of **(x,y)**.



•According to the operations on the image pixels, it can be further divided into 2 categories:

○Point operations and

○Spatial operations (including linear and non-linear operations).

Enhancement Methods

1.Contrast stretching :

•Low-contrast images can result from poor illumination, lack of dynamic range in the image sensor, or even wrong setting of a lens aperture.

•The idea behind contrast stretching is to increase the dynamic range of the gray levels in the image being processed.

• The general form is:

$$s = \frac{1}{1+(m/r)^E}$$

where, r are the input image values, s are the output image values, m is the thresholding value and E the slope.

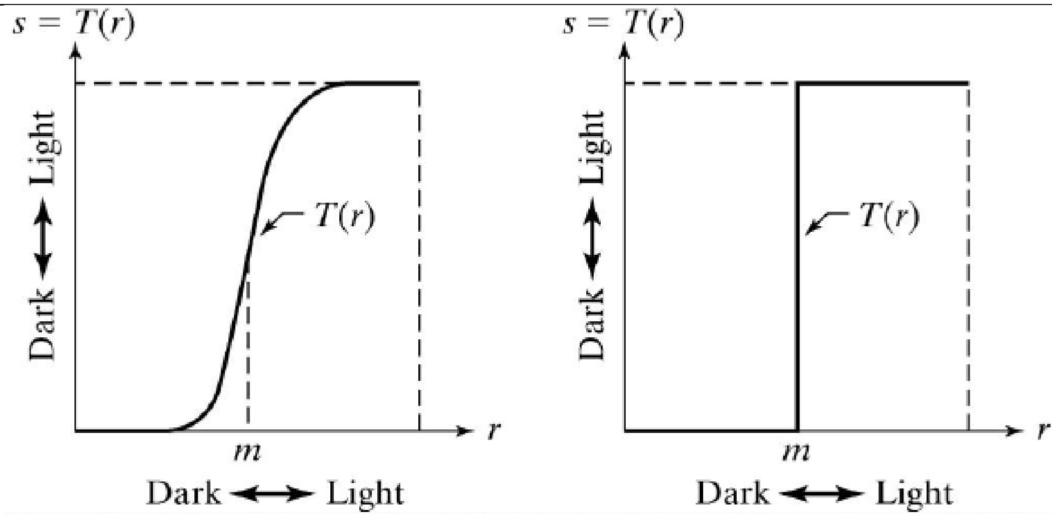


Figure shows the effect of the variable E:

- If E = 1 the stretching became a threshold transformation.
- If E > 1 the transformation is defined by the curve which is smoother and
- When E < 1 the transformation makes the negative and also stretching.

Noise Reduction

This is accomplished by averaging and median filtering. These are as follows:

a. Median Filtering :

- The median filter is normally used to reduce noise in an image by preserving useful detail in the image.
- The median filter considers each pixel in the image in turn and looks at its nearby neighbors to decide whether or not it is representative of its surroundings.
- The median is calculated by first sorting all the pixel values from the surrounding neighborhood into numerical order and then replacing the pixel being considered with the middle pixel value.

Figure below illustrates an example calculation.

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighbourhood values:

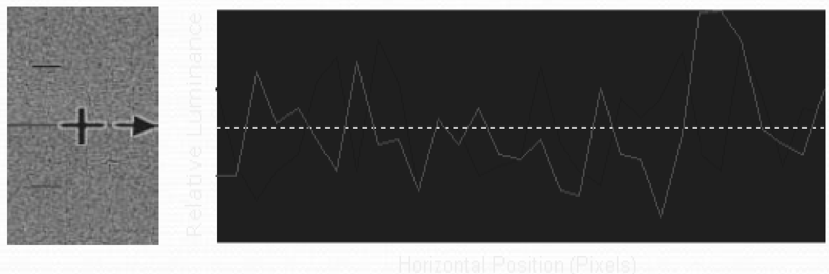
115, 119, 120, 123, 124, 125, 126, 127, 150

Median value: 124

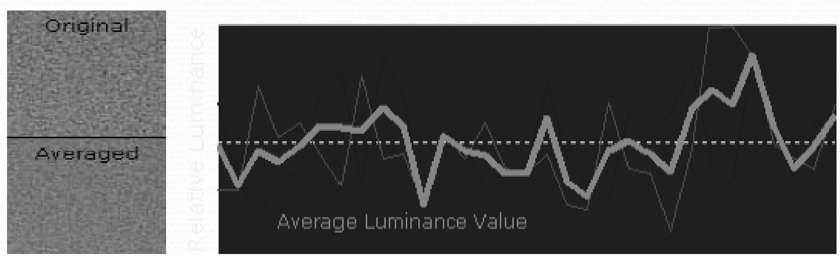
b.Noise removal using Averaging:

- Image averaging works on the assumption that the noise in your image is truly random.
- This way, random fluctuations above and below actual image data will gradually even out as one averages more and more images.

•If you were to take two shots of a smooth gray patch, using the same camera settings and under identical conditions (temperature, lighting, etc.), then you would obtain images similar to those shown on the left.



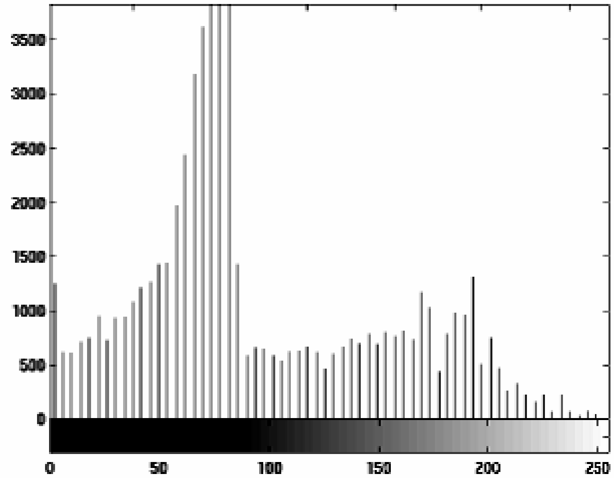
•If we were to take the pixel value at each location along the dashed line, and average it with value for the pixel in the same location for the other image, then the brightness variation would be reduced as follows:



Intensity Adjustment

- Intensity adjustment is a technique for mapping an image's intensity values to a new range.
- For example, rice.tif. is a low contrast image. The histogram of rice.tif, shown in Figure below, indicates that there are no values below 40 or above 225. If you remap the data values to fill the entire intensity range [0, 255], you can increase the contrast of the image.
- You can do this kind of adjustment with the imadjust function. The general syntax of imadjust is

  J = imadjust(I,[low_in high_in],[low_out high_out])



Histogram Equalization

- Histogram Equalization is a technique that generates a gray map which changes the histogram of an image and redistributing all pixels values to be as close as possible to a user – specified desired histogram.
- It allows for areas of lower local contrast to gain a higher contrast.

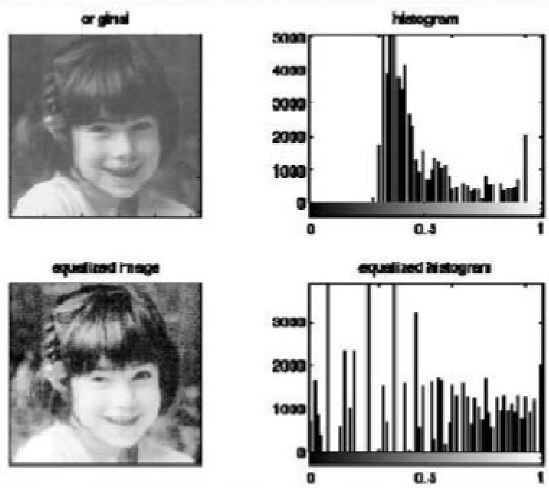


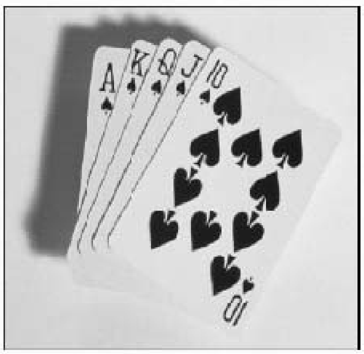
Figure above shows the original image and its histogram, and the equalized versions. Both images are quantized to 64grey levels.

Image Thresholding

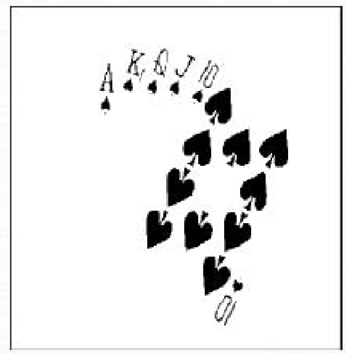
- Thresholding is the simplest segmentation method.
- The pixels are partitioned depending on their intensity value T.
- Global thresholding, using an appropriate threshold T:

$$g(x, y) = \begin{cases} 1, & \text{if } f(x, y) > T \\ 0, & \text{if } f(x, y) <= T \end{cases}$$

- Imagine a poker playing robot that needs to visually interpret the cards in its hand:

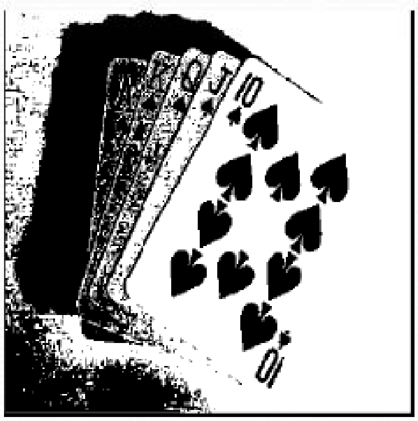


Original Image

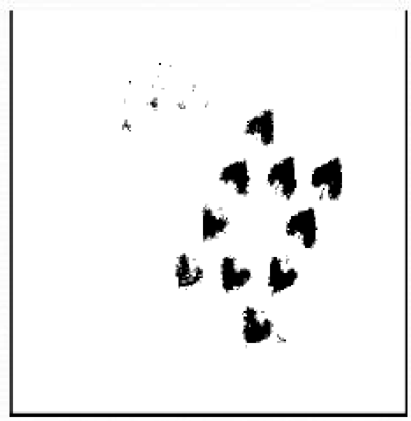


Thresholded Image

If you get the threshold wrong the results can be disastrous:



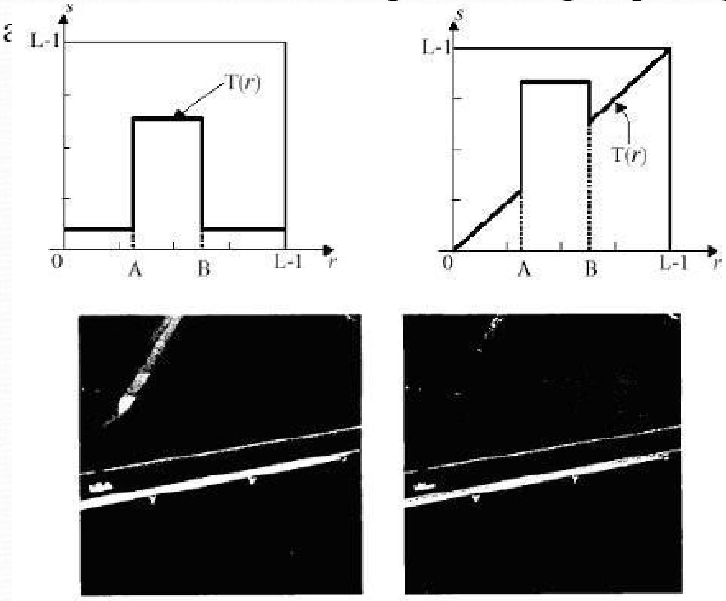
Threshold Too High



Threshold Too Low

Gray Level Slicing

- Grey level slicing is the spatial domain equivalent to band-pass filtering.
- A grey level slicing function can either emphasize a group of intensities and diminish all others or it can emphasize a group of grey levels and leave the rest a



The figure above shows An example of gray level slicing with and without background

Image Rotation

- Image rotation in the digital domain is a form of re-sampling but is performed on non-integer points.

•The equation below gives the coordinate transformation in terms of rotation of the coordinate axis.

$$Sx = Dx\cos(\theta) + Dy\sin(\theta)$$
$$Sy = -Dx\sin(\theta) + Dy\cos(\theta)$$

Where, S and D represent source and destination coordinates.

0° rotation 90° rotation 180° rotation

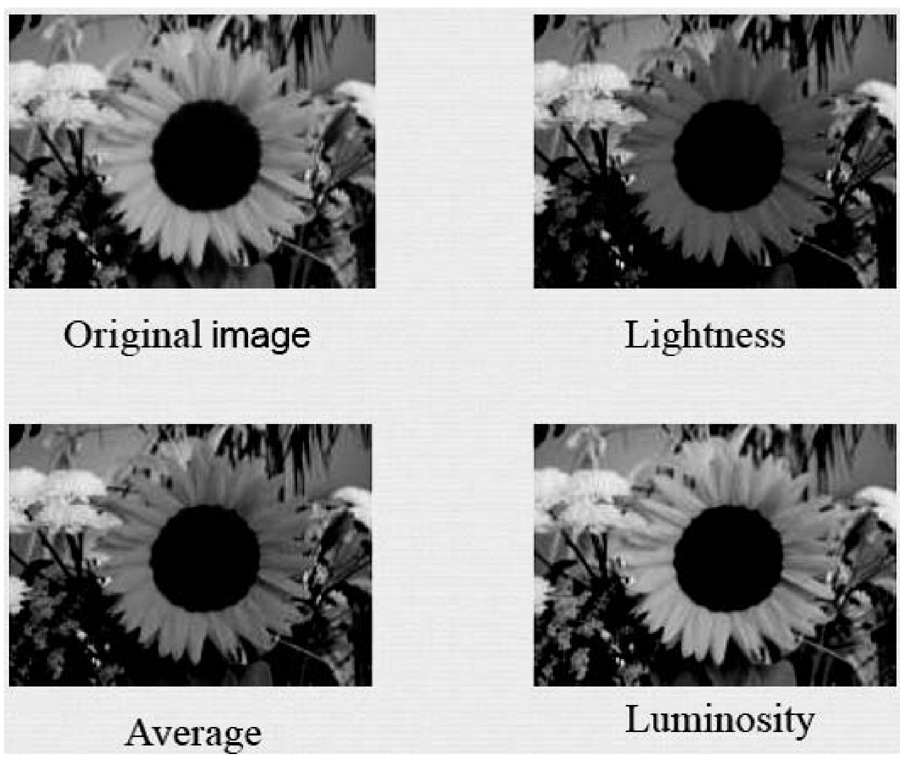
Conversion Methods

1.Greyscale conversion:

•Conversion of a colour image into a greyscale image inclusive of salient features is a complicated process.
•The converted greyscale image may lose contrasts, sharpness, shadow, and structure of the colour image.
•To preserve these salient features, the colour image is converted into greyscale image using three algorithms as stated:

a. The **lightness** method averages the most prominent and least prominent colors: (max(R, G, B) + min(R, G, B)) / 2.
b. The **average** method simply averages the values: (R + G + B) / 3.
c. The **luminosity** method is a more sophisticated version of the average method. The formula for luminosity is 0.21 R + 0.71 G + 0.07 B.

Examples:



2. Image File Format:

- The file format is critical to the preservation of an image.
- The TIFF file (tagged image file format) is the current preservation format because it holds all the preservation information required to create a digital master of the original.

Some of the file formats are: TIFF Preferred Archival format, JPEG Irreversible image compression, DNG Universal camera raw format etc.

Original

JPEG Compression

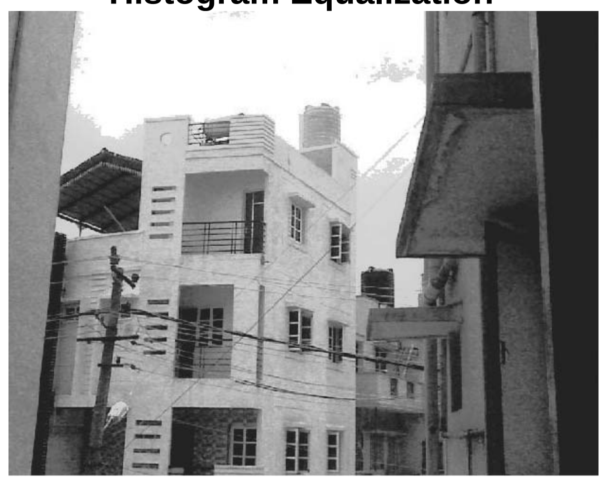
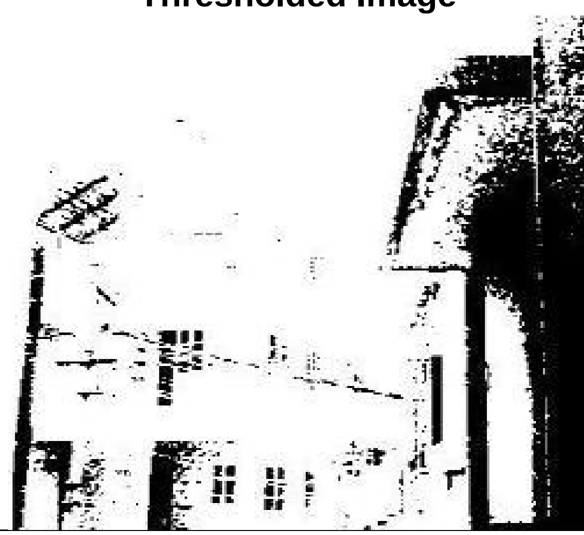
Resources Required

Software requirements:

1. Windows Operating System XP and above.
2. MATLAB 7.10.0(R2010a)

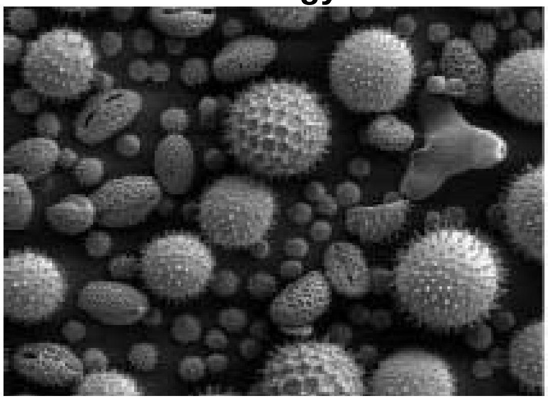
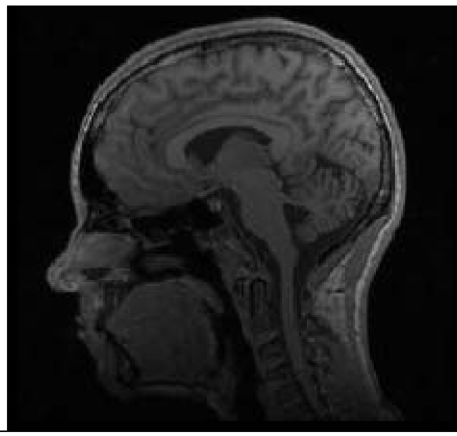
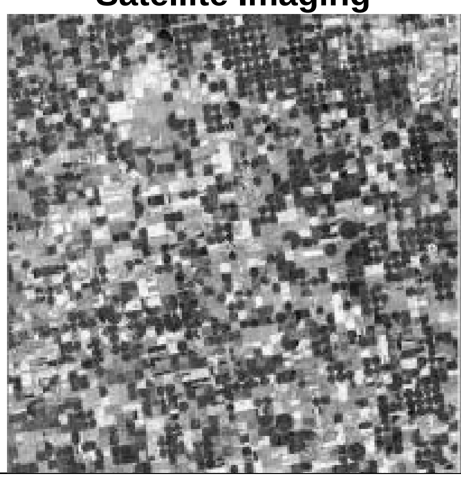
Hardware requirements:

1. Hard disk: 16GB and above.
2. RAM: 1GB and above.
3. Processor: Dual-core and above.

Examples	
Original Image 	**Image with Salt & Pepper Noise** 
Filtered Image 	**Histogram Equalization** 
Contrast Stretched Image 	**Thresholded Image** 

Grayscale slicing background	Grayscale slicing without background
	

180 degree rotation

APPLICATIONS	
Biology 	**Astronomy** 
Medicines 	**Security, Biometrics** 
Satellite Imaging 	**Personal Imagery** 

APPLICATIONS

- In single-rate DSP systems, all data is sampled at the same rate
  - no change of rate within the system.
- In multirate DSP systems, sample rates are changed (or are different) within the system
- Multirate can offer several advantages
  - reduced computational complexity
  - reduced transmission data rate.

Example : Audio Sample Rate Conversion

- recording studios use 192 kHz
- CD uses 44.1 kHz
- wideband speech coding using 16 kHz

- master from studio must be rate-converted by a factor

$$\frac{44.1}{192}$$

Example : Oversampling ADC

Consider a Nyquist rate ADC in which the signal is sampled at the desired precision and at a rate such that Nyquist's sampling criterion is just satisfied.

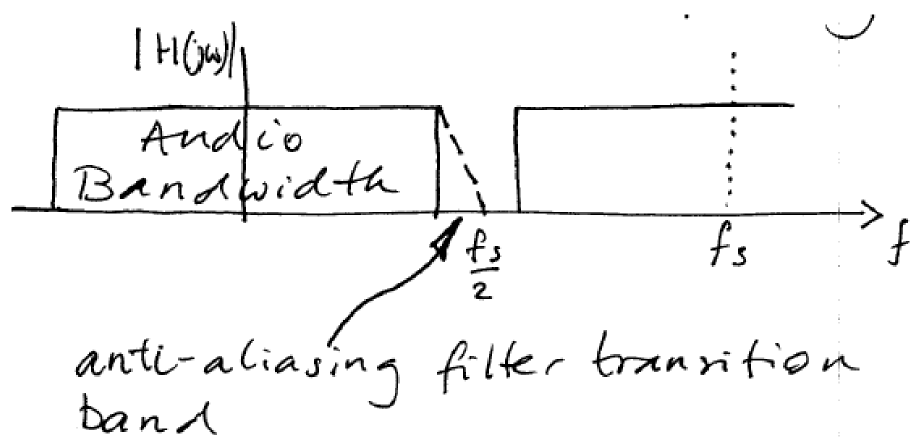
- Bandwidth for audio is 20 Hz $< f <$ 20 kHz
- Antialiasing filter required has very demanding specification

$$|H(j\omega)| = 0 \text{ dB}, f < 20\text{kHz}$$

$$|H(j\omega)| < 96 \text{ dB}, f \geq \frac{44.1}{2}\text{kHz}$$

- Requires high order analogue filter such as elliptic filters that have very nonlinear phase characteristics
  - hard to design, expensive and bad for audio quality.

Nyquist Rate Conversion Anti-aliasing Filter.



Consider oversampling the signal at, say, 64 times the Nyquist rate but with lower precision. Then use multirate techniques to convert sample rate back to 44.1 kHz with full precision.

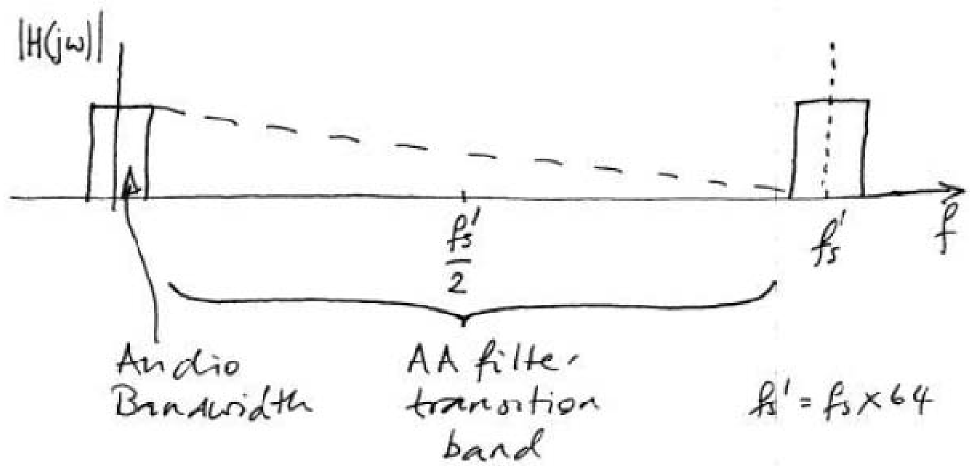
- New (over-sampled) sampling rate is 44.1 × 64 kHz.
- Requires simple antialiasing filter

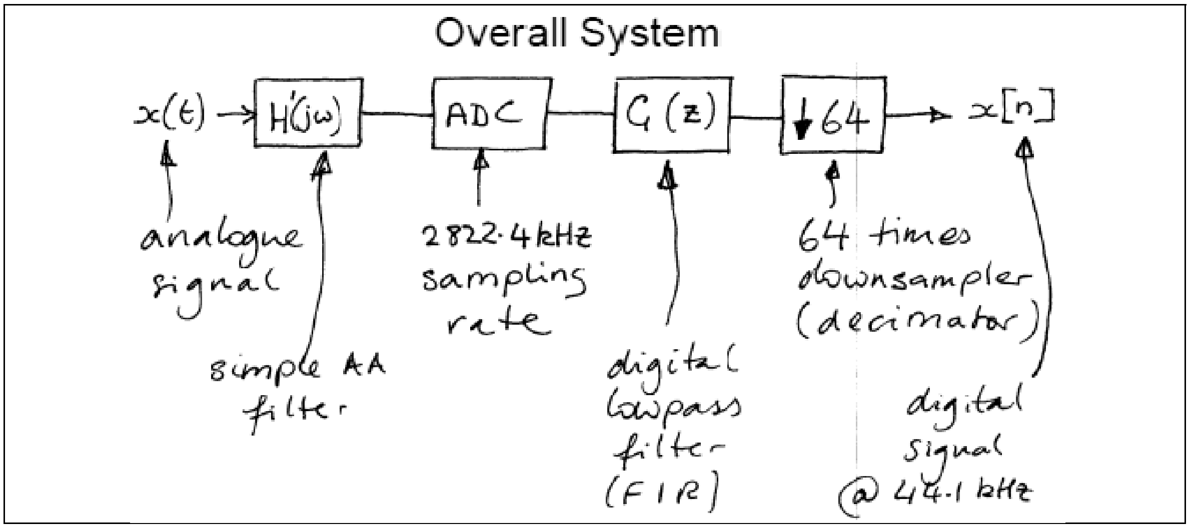
$$|H(j\omega)| = 0 \text{ dB}, f < 20\text{kHz}$$

$$|H(j\omega)| < 96 \text{ dB}, f \geq (44.1 \times 64) - \frac{44.1}{2} \text{kHz}$$

- Could be implemented by simple filter (eg. RC network)
- Recover desired sampling rate by downsampling process.

Oversampled Conversion Antialiasing Filter




Overall System
x(t)
H'(jω)
ADC
G (z)
↓64
x[n]
analogue signal
simple AA filter
2822.4kHz sampling rate
digital lowpass filter (FIR)
64 times downsampler (decimator)
digital signal @ 44.1 kHz

Question Bank.

Unit - I.

1. What do you understand by the terms: Signal and signal processing.
2. What is a Deterministic signal? Give an example.
3. What is random signal?
4. Define (a) Periodic signal (b) Non - periodic signal.
5. Define symmetric and antisymmetric signals.
6. What are energy and power signal?
7. Define the following: (i) Analog signal
   (ii) Discrete- time signal.
   (iii) Digital signal.
8. What are the different types of signal Representation?
9. Define the following: (i) System
   (ii) Discrete-time system.
10. What are the classification of discrete - time systems.
11. What are the different types of operations performed on discrete time signals.
12. What is a shift - invariant system?
13. What is a causal system? Give an example.
14. Define linear system and give example.
15. Define Static and dynamic system.
16. Define a stable and causal systems.
17. What is an LTI system?

18. What is causality condition for an LTI system?

19. What is the condition for system stability.

20. What do you understand by linear convolution?

21. What are the properties of convolution.

22. What is the property of recursive and non-recursive systems?

23. A causal system is one whose impulse response $h(n) = 0$ for $n < 0$. True / False.

24. A linear system is stable if its impulse response is absolutely summable. True / False.

25. A recursive system described by a linear constant difference equation is linear and time-invariant. True / False.

26. How can you find the step response of a system if the impulse response $h(n)$ is known?

27. Determine the unit step response of the LTI system with impulse response.

28. Define the frequency response of a discrete-time system.

29. Obtain the frequency response of discrete-time system with impulse function a) $h(n) = b^n u(n)$ for $|b| < 1$

b) $h(n) = (0.3)^n$ for $n \geq 0$.

30. Explain the linear property of DTFT.

Unit - II

1. The N-pt DFT of a sequence $x(n)$ is ________.
2. The N-pt IDFT of a sequence $X(k)$ is ________.
3. List any four properties of DFT.
4. If $X(k)$ is DFT of a sequence $x(n)$ then DFT of real part of $x(n)$ is ________.
5. For a real valued sequence $x(n)$,
   $X_R(k) =$ ________, $X_I(k) =$ ________.
6. Compute the DFT of $x(n) = \delta(n-n_0)$.
7. Find the DFT of the following signals (i) $x(n) = \delta(n)$ (ii) $x(n) = a^n$.
8. Calculate the DFT of a sequence $x(n) = \left[\frac{1}{4}\right]^n$ for $N=16$
9. State and prove time shifting property of DFT.
10. Find the DFT of the sequence $x(n) = \{1, 1, 0, 0\}$
11. Compute the DFT of the sequence whose are
    $x(n) = \{1, 1, -2, -2\}$.
12. Find the IDFT of $Y(k) = \{1, 0, 1, 0\}$.

13. If X(k) is DFT of a sequence x(n), then DFT of imaginary part of x(n) is ________ .

14. When the DFT X(k) of a sequence x(n) is imaginary.

15. When the DFT X(k) of a sequence x(n) is real.

16. Establish the relation between DFT and Z-transform)

17. Explain circular shifting property of DFT.

18. What is Zero Padding? What are its uses?

19. Define: discrete Fourier series.

20. What do you understand by periodic convolution?

21. Define: circular convolution.

22. How the circular convolution is obtained (steps).

23. Distinguish between linear and circular convolution of two sequences.

24. Obtain the circular convolution of the following sequences $x(n) = \{1, 2, 1\}$; $h(n) = \{1, -2, 2\}$.

25. How will you obtain linear convolution from circular convolution.

26. Obtain linear convolution of the sequences $x(n) = \{1, 2, 3\}$; $h(n) = \{-1, -2\}$ using circular convolution

27. Write a brief note on sectioned convolution.

28. What are the two methods used for the sectional convolution?

29. Write briefly about overlap-save method?

30. Write briefly about overlap-add method.

31. State the differences between overlap save and overlap add method.

32. Distinguish between DFT and DTFT.

33. Distinguish between Fourier series and fourier transform.

Unit - III (IIR filters)

1. Give any two properties of Butterworth lowpass filters.

2. What are the properties of chebyshev filter?

3. Give the equation for the order of N and cutoff frequency Ω of Butterworth filter.

4. Poles of Butterworth filter lie on an ellipse True/False.

5. Poles of chebyshev filter lie on an ellipse True/False.

6. chebyshev filter poles are close to $j\Omega$ axis than those in Butterworth Filter True/False.

7. A causal and stable IIR filter cannot have linear phase True | False.

8. Give the equation for the order N, major, minor and axis of an ellipse in case of chebyshev filter.

9. What are the parameters that can be obtained from the chebyshev filter?

10. Distinguish between Butterworth and chebyshev Type-I filter.

11. How one can design digital filters from analog filters?

12. Mention any two procedures for digitizing the transfer function of an analog filter.

13. What is meant by impulse invariant method of designing IIR filter?

14. Why impulse invariant method is not preferred in the design of IIR filter other than Low pass filter?

15. What is bilinear transformation?

16. What are the properties of bilinear transformations?

17. What is Warping effect? What ~~ots~~ is its effect on magnitude and phase response?

18. Write a short note on Prewarping.

19. What are the advantages and disadvantages of bilinear transformation.

20. Distinguish between recursive realization and non-recursive realization.

FIR Filters:

1. What are the different types of filters based on impulse response?

2. What are the different types of filters based on frequency response?

3. What are the desirable and undesirable features of FIR filters.

4. Distinguish between IIR and FIR.

5. What are the techniques of designing FIR filter.

6. What do you understand by linear phase response?

7. What is the reason that FIR filter is always stable?

8. State the condition for a digital filter to be causal and stable.

9. What are the properties of FIR filter?

10. How the zeros in FIR filter is located? Explain briefly.

11. What is the basis for Fourier series method of design? Why truncation is necessary?

12. Explain briefly the method of designing FIR filter using Fourier series method.

13. What are the disadvantages of Fourier series method?

14. What are Gibbs oscillations?

15. Explain the ~~prodo~~ procedure for designing FIR filters using windows.

16. What are the desirable characteristics of the windows.

17. What is the principle of designing FIR filter using window

18. What is window? why it is necessary?

19. Give the equation specifying Hanning and Blackman windows.

20. Give the equation specifying Barlett and Hamming windows.

21. Give the expression for the frequency response of
a) Barlett window b) Blackman window.

22. Give the equation specifying kaiser window.

23. What are the advantages of kaiser window.

24. What is the principle of designing FIR filter using frequency sampling method?

25. Compare Hamming with kaiser window.

Unit - IV

1. What are the different types of arithmetic in digital system?

2. What do you understand by a fixed-point number?

3. What are the different types in fixed point number representation?

4. What do you understand by sign-magnitude representation?

5. Write a short notes on 1's complement representation?

6. What do you understand by 2's complement representation?

7. Write an account on floating point arithmetic.

8. What is meant by block floating point representation? What are its advantages?

9. What are the advantages of floating point arithmetic?

10. Compare the fixed point and floating point arithmetic.

11. What are the three quantization errors due to finite word length registers to digital filters?

12. How the multiplications and additions are carried out in floating point arithmetic.

13. Brief on coefficient accuracy.

14. What is product round off error in digital signal processing?.

15. What do you understand by Input quantization error?

16. What are the different quantization methods?

17. What is truncation? What is the error that arises due to truncation in floating point numbers?

18. What is the relationship between truncation error 'e' and the bits 'B' for representing a decimal into binary?

19. What is meant by rounding? Discuss its effect on all types of number representation.

20. What is meant by A/D conversion noise?

21. What is the effect of quantization on pole locations?

22. What is meant by quantization step size?

23. What is meant by limit cycle oscillations?

24. What is overflow oscillations?

25. What are the methods used to prevent overflow?

26. What is meant by saturation anthrietic, what is its disadvantages?

27. What are the two kinds of limit cycle behaviours in DSP?

28. Determine 'dead band' of the filter.

29. Give the expression for signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit.

30. Why rounding is Preferred to truncation in realizing digital filter?

Unit-V

1. What is multirate signal processing?
2. What is the importance of 'D' Factor in multi multirate signal processing?
3. How do you reduce the sampling rate?
4. What is the use of upsampling of a signal?
5. How do you acheive sampling rate conversion?
6. Write a note on downsampling process. Give an example.
7. Describe the importance of polyphase filters in multirate signal processing.
8. What are the applications of multirate DSP?
9. What is speech compression? why we need?
10. Define (i) PCM (ii) DPCM.
11. Explain the operation of ADPCM with a neat diagram.
12. What is LPC?
13. Explain in detail about LPC decoder.
14. ~~Why we~~ What is the need of Adaptive filters? What are its advantages?
15. What are the applications of Adaptive filter?

16. What is Single Echo filter?
17. What is Multiple Echo filter?
18. What is Image Enhancement?
19. Why we need Image enhancement?
20. What is Image procesing.
21. What are the techniques of Image enhancement?
22. Explain the Enhancement methods with an example.
23. What are the applications of ~~mostticals~~ musical sound processing?.
24. What are the advantages of multirate signal Processing?
25. Explain the application of Multirate DSP — Oversampling ADC.

—x—.